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FORCE AND MOMENT COEFFICIENTS FOR A THIN AIRFOIL WITH FLAP AND TAB IN A FORM USEFUL FOR STABILITY AND CONTROL CALCULATIONS

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SUMMARY

Recent airplane design trends have been directed along the line of control-free-stability analysis involving motions of the control surfaces having frequencies less than those considered in flutter calculations, but great enough to require a knowledge of the air forces resulting from velocity displacements. In addition to this, control arrangements have been considered having the entire airfoil pivoted or having a flap or tab actuated by pressures developed on the contour of the airfoil. In order to facilitate these calculations a systematic presentation of the necessary pressure and force coefficients acting on a thin airfoil having a flap and tab has been made and presented in this report in a form suitable for stability calculations.

Equations for the pressures have been derived from the basic equations of reference 1 and their development has been given in detail. Force coefficients have been taken directly from references 1 and 2 or integrated graphically from the curves of pressure distribution. In order to preserve a continuity of equations, the displacement, velocity, and acceleration stability coefficients all have been calculated even though the acceleration or virtual mass effect generally may be neglected in stability calculations. In all cases only the real part of the complex equations of reference 1 is considered due to the order of frequencies generally involved.

SYMBOLS

α angle of attack referred to relative wind (radians)

θ,α' angle of rotation of wing referred to horizontal (radians)

$-\frac{\dot{h}}{v},\gamma$ angle between the relative wind vector and horizontal (radians)

δ,β flap deflection (radians)

δ_t tab deflection (radians)

b semi-chord of airfoil (ft)

c flap hinge position in fractions of the airfoil semi-chord (measured from airfoil center)

a center of rotation of airfoil in fractions of the airfoil semi-chord (measured from airfoil center)

H center of rotation of airfoil in fractions of the total chord (measured from L.E.)

E ratio of the flap chord to the total chord

c_T total chord of airfoil (ft)

c_f flap chord (ft)

c_t tab chord (ft)

h vertical displacement of airfoil (ft)

v velocity (fps)

M.A.C. mean aerodynamic chord (ft)

x distance from airfoil chord midpoint to any chordwise station measured in fractions of the semi-chord

p pressure difference between upper and lower surfaces on airfoil

t time (sec)

ρ mass air density (slugs/ft^3)

φ velocity potential function

Γ circulation

$P = p \Big/ \frac{1}{2}\rho v^2$

C_L lift coefficient

C_m pitching moment coefficient

C_H hinge moment coefficient

$C_{L()} = \dfrac{\partial C_L}{\partial ()}$

$C_{m()} = \dfrac{\partial C_m}{\partial ()}$

$C_{H()} = \dfrac{\partial C_H}{\partial ()}$

$P_{()} = \dfrac{\partial P}{\partial ()}$

NOTATION

Before the stability coefficients may be derived from the equations of reference 1 it is necessary to distinguish between the degrees of freedom employed in reference 1 and those generally used in control-free-stability calculations.

Consider an airfoil to have its center of motion position at point a as shown in figure 1. The angle θ as shown in this figure will define the angle of the chord with respect to the horizontal and the angle γ will be the angle of the flight path with respect to the horizontal. Then

$$\theta = \alpha' \qquad \gamma = -\frac{\dot{h}}{v}$$

where α' denotes the value of α as used in reference 1 to distinguish from the value of α employed in stability calculations.

The angle of attack of the airfoil is then

$$\alpha = \theta - \gamma = \alpha' + \frac{\dot{h}}{v}$$

where $\dot{h}$ is the translational velocity defined in reference 1. In the stability notation chosen the differential operator $D()$ will indicate $\partial()/\partial\tau$ where τ is the aerodynamic time having units of half-chord lengths. The value of τ is defined by the equation $\tau b = vt$, giving

$$\partial\tau/\partial t = v/b$$

where

v true airspeed (ft/sec)

t time (sec)

b half chord of airfoil (ft)

The degrees of freedom of the airfoil will be α, θ, δ, and δ_t where δ is the flap angle and is equivalent to β used in reference 1, and δ_t is the tab angle. All angles will be expressed in radian measure.

In forming the stability coefficients each motion is assumed to take place with the other three degrees of freedoms maintaining constant values. It is now necessary to replace various terms of the equations given in reference 1 with the newly defined coordinates. The substitutions are given in table I and are all apparent with the exception of those for $D(\theta)$ and $D^2(\theta)$.

The value of $D(\theta)$ expresses the airfoil rotating at uniform angular velocity but maintaining a constant angle of attack α. To accomplish this the flight path angle must have a constant rate of change producing a curved flight path; hence

$$\dot{\theta} = \frac{\partial\theta}{\partial\tau}\frac{\partial\tau}{\partial t} = D(\theta)\ \frac{v}{b}$$

giving

$$D(\theta) = \frac{b\dot{\theta}}{v} = \frac{b\dot{\alpha}'}{v}$$

$$D^2(\theta) = \frac{b^2\ddot{\alpha}'}{v^2}$$

If α is to be constant then

$$\alpha' = \theta = -\frac{\dot{h}}{V} + \text{constant}$$

giving

$$\dot{\theta} = -\frac{\ddot{h}}{V} \quad \text{or} \quad D(\theta)\,\frac{V}{b} = -\frac{\ddot{h}}{V}$$

hence

$$-D(\theta) = \frac{b\ddot{h}}{V^2}$$

In order to obtain equations in terms of $D(\theta)$, a substitution for $\dot{\alpha}'$ and $\ddot{h}$ from the equations of reference 1 must be made.

In case the airfoil considered is a tail plane the aerodynamic time τ will be based upon the main wing chord which may be the wing mean aerodynamic chord. In this case the value of a is to be measured in terms of half-chord lengths of the tail chord and the final value of the stability or pressure coefficient converted by multiplying by C_T/M.A.C. for $D()$ and $(C_T/\text{M.A.C.})^2$ for $D^2()$ coefficients, where C_T is the tail chord. If l is the distance of the aerodynamic center of the tail plane aft of the airplane center of gravity in feet, then

$$a = -\frac{2l}{C_T} - \frac{1}{2}$$

The stability and pressure coefficients have the subscript denoting the type of coefficient; for example,

$$P_{D(\theta)} = \frac{\partial P}{\partial D(\theta)}$$

$$C_{H_{D^2(\delta)}} = \frac{\partial C_H}{\partial D^2(\delta)}$$

TABLE FOR CONVERTING EQUATIONS OF REFERENCE 1 TO FORM

EQUATIONS OF THE STABILITY COEFFICIENTS

To obtain an equation in:	Replace the following terms of reference 1*	By
α	$\dot{h}/v$	α
$D(\alpha)$	$b\ddot{h}/v^2$	$D(\alpha)$
$D(\theta)$	$b\dot{\alpha}'/v$ $b\ddot{h}/v^2$	$D(\theta)$ $-D(\theta)$
$D^2(\theta)$	$b^2\ddot{\alpha}'/v^2$	$D^2(\theta)$
δ	β	δ
$D(\delta)$	$b\dot{\beta}/v$	$D(\delta)$
$D^2(\delta)$	$b^2\ddot{\beta}/v^2$	$D^2(\delta)$

*All other terms of equations taken as zero

$$\frac{c - a}{2} = 1 - E - H$$

PRESSURE DISTRIBUTION OVER A THIN AIRFOIL WITH FLAP

The chordwise distribution of pressure difference over a thin airfoil with flap will be calculated using the equations of the non-circulatory and circulatory velocity potentials given in reference 1. It will be assumed that the frequency of oscillation is low enough for terms containing G, as defined in reference 1, to be taken equal to zero. The part of the pressure distribution not containing $2\pi F$ will be defined as the "basic pressure distribution," and that part containing $2\pi F$ will define the additional pressure distribution. This segregation will permit airfoil characteristics to be calculated for airfoils having a finite aspect ratio. From page 6 of reference 1, the pressure difference p between the upper and the lower surface of the airfoil can be written as:

$$\frac{\Delta p}{q} = P = 2\rho\left[\frac{v}{b}\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial t}\right] \tag{1}$$

A positive pressure difference is taken as upward, which is the reverse of reference 1. From page 5 of reference 1, the velocity potential of the non-circulatory flow, using the notation of reference 1 and substituting α' for α, is

$$\varphi = vb\left[\sqrt{1 - x^2}\right]\alpha' + vb\left[\left(\frac{x}{2} - a\right)\sqrt{1 - x^2}\right]\frac{b\dot{\alpha}'}{v}$$

$$+ vb\left[\sqrt{1 - x^2}\right]\frac{\dot{h}}{v} + vb\left[\frac{1}{\pi}\left(\cos^{-1} c\sqrt{1 - x^2} - (x - c)\log N\right)\right]\beta$$

$$+ vb\left[\frac{1}{2\pi}\left(\sqrt{1 - c^2}\sqrt{1 - x^2} + (x - 2c)\sqrt{1 - x^2}\cos^{-1} c\right.\right.$$

$$\left.\left.- (x - c)^2\log N\right)\right]\left(\frac{b\dot{\beta}}{v}\right) \tag{2}$$

where

$$N = \left(1 - cx - \sqrt{1 - x^2}\sqrt{1 - c^2}\right)\Big/(x - c) \tag{3}$$

then

$$\frac{\partial\varphi}{\partial x} = vb\left[\frac{-x}{\sqrt{1 - x^2}}\right]\alpha' + vb\left[\frac{1 + 2ax - 2x^2}{2\sqrt{1 - x^2}}\right]\left(\frac{b\dot{\alpha}'}{v}\right)$$

$$+ vb\left[\frac{-x}{\sqrt{1 - x^2}}\right]\left(\frac{\dot{h}}{v}\right) + vb\left[\frac{1}{\pi}\left(\frac{-x}{\sqrt{1 - x^2}}\cos^{-1} c - \log N\right.\right.$$

$$\left.\left.- (x - c)\frac{\partial \log N}{\partial x}\right)\right]\beta + vb\left[\frac{1}{2\pi}\left(\frac{-x\sqrt{1 - c^2}}{\sqrt{1 - x^2}} + \sqrt{1 - x^2}\cos^{-1} c\right.\right.$$

$$\left.\left.- \frac{x(x - 2c)}{\sqrt{1 - x^2}}\cos^{-1} c - 2(x - c)\log N - (x - c)^2\frac{\partial \log N}{\partial x}\right)\right]\left(\frac{b\dot{\beta}}{v}\right) \tag{4}$$

where

$$\frac{\partial \log N}{\partial x} = \frac{1}{N}\frac{\partial N}{\partial x}$$

$$= \frac{\left(x\sqrt{1 - c^2} - c\sqrt{1 - x^2}\right)}{\sqrt{1 - x^2}\left(1 - cx - \sqrt{1 - x^2}\sqrt{1 - c^2}\right)} - \frac{1}{x - c} \qquad (5)$$

$$\frac{\partial\varphi}{\partial t} = v^2\left[\sqrt{1 - x^2}\right]\frac{b\dot{\alpha}'}{v} + v^2\left[\left(\frac{x}{2} - a\right)\sqrt{1 - x^2}\right]\frac{b^2\ddot{\alpha}'}{v^2}$$

$$+ v^2\left[\sqrt{1 - x^2}\right]\frac{b\ddot{h}}{v^2} + v^2\left[\frac{1}{\pi}\left(\sqrt{1 - x^2}\cos^{-1}c - (x - c)\log N\right)\right]\frac{b\dot{\beta}}{v}$$

$$+ v^2\left[\frac{1}{2\pi}\left(\sqrt{1 - c^2}\sqrt{1 - x^2} + (x - 2c)\sqrt{1 - x^2}\cos^{-1}c\right.\right.$$

$$\left.\left. - (x - c)^2\log N\right)\right]\frac{b^2\ddot{\beta}}{v^2} \qquad (6)$$

If equations (4) and (6) are substituted in equation (1), the pressure distribution for the non-circulatory flow can be found. It is now necessary to determine such a circulatory flow that the pressure between upper and lower surfaces at the trailing edge is zero. If this pressure distribution is added to the above distribution, then the pressure distribution due to the movement of the airfoil having an infinite aspect ratio can be found. Pressures determined by this method will produce a lift; hence the angle of attack of the airfoil will be reduced so that the net lift is zero by deducting the pressure distribution due to α' in proportion to the effective change in angle of attack. The resulting pressure distribution then will be termed the "basic pressure distribution" and will be independent of aspect ratio.

The pressure distribution due to the non-circulatory flow obtained by substituting equations (4) and (6) in equation (1) is

$$p_0 = \frac{\rho}{2} v^2 \left[\frac{-4x}{\sqrt{1 - x^2}}\right] \alpha' + \frac{\rho}{2} v^2 \left[\frac{2(3 + 2ax - 4x^2)}{\sqrt{1 - x^2}}\right] \frac{b\dot{\alpha}'}{v}$$

$$+ \frac{\rho}{2} v^2 \left[4\left(\frac{x}{2} - a\right)\sqrt{1 - x^2}\right] \frac{b^2\ddot{\alpha}'}{v^2} + \frac{\rho}{2} v^2 \left[\frac{-4x}{\sqrt{1 - x^2}}\right] \frac{\dot{h}}{v}$$

$$+ \frac{\rho}{2} v^2 \left[4\sqrt{1 - x^2}\right] \frac{b\ddot{h}}{v^2} + \frac{\rho}{2} v^2 \left[\frac{4}{\pi}\left(\frac{-x \cos^{-1} c}{\sqrt{1 - x^2}} - \log N\right.\right.$$

$$\left.\left. - \frac{(x - c)\, \partial \log N}{\partial x}\right)\right] \beta + \frac{\rho}{2} v^2 \left[\frac{2}{\pi}\left(-x\sqrt{\frac{1 - c^2}{1 - x^2}}\right.\right.$$

$$+ \frac{(3 + 2cx - 4x^2)}{\sqrt{1 - x^2}} \cos^{-1} c - 4(x - c) \log N$$

$$\left.\left. -(x - c)^2 \frac{\partial \log N}{\partial x}\right)\right] \frac{b\dot{\beta}}{v} + \frac{\rho}{2} v^2 \left[\frac{2}{\pi}\left(\sqrt{1 - x^2}\left(\sqrt{1 - c^2}\right.\right.\right.$$

$$\left.\left.\left. - 2c \cos^{-1} c + x \cos^{-1} c\right) - (x - c)^2 \log N\right)\right] \frac{b^2\ddot{\beta}}{v^2} \qquad (7)$$

The pressure distribution due to the circulatory flow neglecting the wake effects can be found by letting $x_0 \to \infty$ in the equation for $(2\pi/\Delta\Gamma)\ \partial\varphi/\partial x$ as given on page 6 of reference 1. Then, when $\Delta\Gamma = \Gamma$, equation (1) gives

$$p_\Gamma = \frac{2\rho v}{b} \left(\frac{\Gamma}{2\pi}\right) \frac{1}{\sqrt{1 - x^2}} \qquad (8)$$

Now for zero pressure at the trailing eige

$$p_0 + p_\Gamma = 0 \quad \text{for} \quad x = 1 \qquad (9)$$

In order to avoid an indeterminate condition at $x = 1$, multiply p_o and p_Γ by $\sqrt{1 - x^2}$ before substituting in equation (9). Then

$$\frac{\Gamma}{2\pi} = bv\left[\alpha' + \left(\frac{1}{2} - a\right)\frac{b\dot{\alpha}'}{v} + \frac{\dot{h}}{v} + \frac{T_{10}}{\pi}\beta + \frac{T_{11}}{2\pi}\frac{b\dot{\beta}}{v^2}\right] \tag{10}$$

where

$$\left.\begin{aligned} T_{10} &= \sqrt{1 - c^2} + \cos^{-1} c \\ T_{11} &= (1 - 2c)\cos^{-1} c + (2 - c)\sqrt{1 - c^2} \end{aligned}\right\} \tag{11}$$

as defined in reference 1. If equation (10) is substituted in equation (8), the pressure distribution becomes

$$p_\Gamma = \frac{4}{\sqrt{1 - x^2}}\left(\frac{\rho}{2}v^2\right)\left[\alpha' + \left(\frac{1}{2} - a\right)\frac{b\dot{\alpha}'}{v} + \frac{\dot{h}}{v} + \frac{T_{10}}{\pi}\beta + \frac{T_{11}}{2\pi}\frac{b\dot{\beta}}{v}\right] \tag{12}$$

The total pressure distribution written in terms of the pressure ratio P for an airfoil of infinite aspect ratio is

$$\begin{aligned} P &= (p_o + p_\Gamma)/\tfrac{\rho}{2}v^2 \\ &= \left[4\sqrt{\frac{1-x}{1+x}}\right]\alpha' + \left[8\sqrt{1-x^2} - 4a\sqrt{\frac{1-x}{1+x}}\right]\frac{b\dot{\alpha}'}{v} + \left[4\left(\frac{x}{2} - a\right)\sqrt{1-x^2}\right]\frac{b^2\ddot{\alpha}'}{v^2} \\ &+ \left[4\sqrt{\frac{1-x}{1+x}}\right]\frac{\dot{h}}{b} + \left[4\sqrt{1-x^2}\right]\frac{b\ddot{h}}{v^2} + \frac{4}{\pi}\left[\sqrt{\frac{1-x}{1+x}}\cos^{-1} c + \sqrt{\frac{1-c^2}{1-x^2}} - \log N\right. \\ &\left. - (x-c)\frac{\partial \log N}{\partial x}\right]\beta + \frac{2}{\pi}\left[(2-c-x)\sqrt{\frac{1-c^2}{1-x^2}} + 4\sqrt{1-x^2} - 2c\sqrt{\frac{1-x}{1+x}}\right. \\ &\left. - 4(x-c)\log N - (x-c)^2\frac{\partial \log N}{\partial x}\right]\frac{b\dot{\beta}}{v} + \frac{2}{\pi}\left[\sqrt{1-x^2}\left(\sqrt{1-c^2}\right.\right. \\ &\left.\left. - (2c-x)\cos^{-1} c\right) - (x-c)^2 \log N\right]\frac{b^2\ddot{\beta}}{v^2} \end{aligned} \tag{13}$$

To form the basic pressure distribution, the angle of attack α' will be altered so that Γ becomes zero. The pressure distribution due to a change in α' only is from equation (13)

$$P_{\alpha}' = 4\sqrt{\frac{1-x}{1+x}}\,\alpha' \tag{14}$$

Now, whenever a virtual or actual change in the airfoil camber occurs due to $\dot{\alpha}'$, β, or $\dot{\beta}$, the angle α' of equation (10) must be changed so that Γ is zero. This must be done since by definition the basic pressure distribution is selected for zero lift. For this condition the following pressure distribution must be deducted from equation (13) to form the basic pressure distribution.

$$P_{\Gamma_0} = 4\sqrt{\frac{1-x}{1+x}}\left[\alpha' + \left(\frac{1}{2} - a\right)\frac{b\dot{\alpha}'}{v} + \frac{\dot{h}}{v} + \frac{T_{10}}{\pi}\beta + \frac{T_{11}}{2\pi}\frac{b\dot{\beta}}{v}\right] \tag{15}$$

then the basic pressure distribution which will occur for $\Gamma = 0$ is

$$P_B = P - P_{\Gamma_0}$$

$$= \left[8\sqrt{1-x^2} - 2\sqrt{\frac{1-x}{1+x}}\right]\frac{b\dot{\alpha}'}{v} + \left[4\left(\frac{x}{2}-a\right)\sqrt{1-x^2}\right]\frac{b^2\ddot{\alpha}'}{v^2} + \left[4\sqrt{1-x^2}\right]\frac{b\ddot{h}}{v^2}$$

$$+ \frac{4}{\pi}\left[x\sqrt{\frac{1-c^2}{1-x^2}} - \log N - (x-c)\frac{\partial \log N}{\partial x}\right]\beta + \frac{2}{\pi}\left[\frac{(3+x-4x^2)}{\sqrt{1-x^2}}\cos^{-1} c\right.$$

$$\left. + (1-c)x\sqrt{\frac{1-c^2}{1-x^2}} - 4(x-c)\log N - (x-c)^2\frac{\partial \log N}{\partial x}\right]\frac{b\dot{\beta}}{v}$$

$$+ \frac{2}{\pi}\left[\sqrt{1-x^2}\left(\sqrt{1-c^2} + (x-2c)\cos^{-1} c\right) - (x-c)^2 \log N\right]\frac{b^2\ddot{\beta}}{v^2} \tag{16}$$

For a wing having a finite span the slope of the lift curve m will be less than 2π. This will cause P_{α}' from equation (14) to be reduced by the factor $\frac{m}{2\pi}$. If the lift effectiveness due to the change in camber is defined as

$$K_{\dot{\alpha}'} = \left(\frac{1}{2} - a\right), \quad K_{\beta} = \frac{T_{10}}{\pi}, \quad K_{\dot{\beta}} = \frac{T_{11}}{2\pi} \tag{17}$$

and

$$P_A = \frac{2}{\pi}\sqrt{\frac{1 - x}{1 + x}} \tag{18}$$

then the total pressure ratio for a finite wing having a slope of lift curve m can be written as

$$\begin{aligned}
P = {} & \left[mP_A\right] \alpha' + \left[P_{B\dot{\alpha}'} + mK_{\dot{\alpha}} P_A\right] \frac{b\dot{\alpha}'}{V} + \left[P_{B\ddot{\alpha}'} - aP_{B\ddot{h}}\right] \frac{b^2\ddot{\alpha}'}{V^2} \\
& + \left[mP_A\right] \frac{\dot{h}}{V} + \left[P_{B\ddot{h}}\right] \frac{b\ddot{h}}{V^2} + \left[P_{B\beta} + mK_{\beta} P_A\right] \beta + \left[P_{B\dot{\beta}} + mK_{\dot{\beta}} P_A\right] \frac{b\dot{\beta}}{V} \\
& + \left[P_{B\ddot{\beta}}\right] \frac{b^2\ddot{\beta}}{V^2}
\end{aligned} \tag{19}$$

where

$$\left.\begin{aligned}
P_{B\dot{\alpha}'} &= 8\sqrt{1-x^2} - 2\sqrt{\frac{1-x}{1+x}} \\
P_{B\ddot{\alpha}'} &= 2x\sqrt{1-x^2} \\
P_{B\ddot{h}} &= 4\sqrt{1-x^2} \\
P_{B\beta} &= \frac{4}{\pi}\left[x\sqrt{\frac{1-c^2}{1-x^2}} - \log N - (x-c)\frac{\partial \log N}{\partial x}\right] \\
P_{B\dot{\beta}} &= \frac{2}{\pi}\left[\frac{(3+x-4x^2)}{\sqrt{1-x^2}} \cos^{-1} c + (1-c)x\sqrt{\frac{1-c^2}{1-x^2}}\right. \\
&\qquad \left. - 4(x-c) \log N - (x-c)^2 \frac{\partial \log N}{\partial x}\right] \\
P_{B\ddot{\beta}} &= \frac{2}{\pi}\left[\sqrt{1-x^2}\left(\sqrt{1-c^2} + (x-2c)\cos^{-1} c\right) - (x-c)^2 \log N\right]
\end{aligned}\right\} \tag{20}$$

Values of the coefficients defined by equations (17), (18), and (20) have been calculated and plotted in figures 3 to 7.

LIFT, PITCHING MOMENT, AND FLAP HINGE MOMENT FOR A THIN AIRFOIL WITH A FLAP

From the equations for the pressure distribution determined in the previous section, the airfoil lift and moment coefficients can be obtained by integration. These integrations have been performed in reference (1) for the case of the airfoil with a single flap. Hence, it will only be necessary to rewrite the equations here using the necessary notations to convert to coefficient form. Let

$$C_L = \frac{-P}{\rho\ b\ v^2}$$

$$C_m = \frac{M_\alpha}{2\ \rho\ b^2\ v^2}$$

$$C_H = \frac{M_\beta}{2\ \rho\ v^2\ b^2\ E^2}$$

where

P lift force given by equation (XVIII) of reference 1

M_α pitching moment given by equation (XX) of reference 1

M_β hinge moment given by equation (XIX) of reference 1

E ratio of flap chord to total chord

$2\pi F = m$, slope of airfoil lift curve

then

$$C_L = m\alpha' + \left[\pi + \left(\frac{1}{2} - a\right) m\right] \frac{b\dot{\alpha}'}{v} + \left[-a\pi\right] \frac{b^2\ddot{\alpha}'}{v^2} + \left[m\right] \frac{\dot{h}}{v} + \left[\pi\right] \frac{b\ddot{h}}{v^2}$$

$$+ \left[m\ \frac{T_{10}}{\pi}\right] \beta + \left[-T_4 + m\ \frac{T_{11}}{2\pi}\right] \frac{b\dot{\beta}}{v} + \left[-T_1\right] \frac{b^2\ddot{\beta}}{v^2} \qquad (21)$$

$$C_m = \left[m\left(\frac{a}{2} + \frac{1}{4}\right)\right]\alpha' + \left[-\frac{\pi}{2}\left(\frac{1}{2} - a\right) + m\,\frac{1}{2}\left(\frac{1}{4} - a^2\right)\right]\frac{b\dot{\alpha}'}{V}$$

$$+ \left[-\frac{\pi}{2}\left(\frac{1}{8} + a^2\right)\right]\frac{b^2\ddot{\alpha}'}{V^2} + \left[m\left(\frac{a}{2} + \frac{1}{4}\right)\right]\frac{\dot{h}}{V} + \left[\frac{a\pi}{2}\right]\frac{b\ddot{h}}{V^2}$$

$$+ \left[-\frac{(T_4+T_{10})}{2} + m\left(\frac{a}{2}+\frac{1}{4}\right)\frac{T_{10}}{\pi}\right]\beta + \left[-\frac{1}{2}\left(T_1 - T_8 - (c-a)T_4 + \frac{T_{11}}{2}\right)\right.$$

$$\left. + m\left(\frac{a}{2} + \frac{1}{4}\right)\frac{T_{11}}{2\pi}\right]\frac{b\dot{\beta}}{V} + \left[\frac{T_7}{2} + (c-a)\,\frac{T_1}{2}\right]\frac{b^2\ddot{\beta}}{V^2} \tag{22}$$

$$C_H = \frac{1}{E^2}\left[-m\,\frac{T_{12}}{4\pi}\right]\alpha' + \frac{1}{E^2}\left[T_9 + \frac{T_1}{2} - T_4\left(\frac{a}{2} - \frac{1}{4}\right) - m\left(\frac{1}{2} - a\right)\frac{T_{12}}{4\pi}\right]\frac{b\dot{\alpha}'}{V}$$

$$+ \frac{1}{E^2}\left[-T_{13}\right]\frac{b^2\ddot{\alpha}'}{V^2} + \frac{1}{E^2}\left[-m\,\frac{T_{12}}{4\pi}\right]\frac{\dot{h}}{V} + \frac{1}{E^2}\left[\frac{T_1}{2}\right]\frac{b\ddot{h}}{V^2}$$

$$+ \frac{1}{E^2}\left[-\frac{1}{2\pi}\left(T_5 - T_4T_{10}\right) - m\,\frac{T_{12}}{4\pi}\,\frac{T_{10}}{\pi}\right]\beta$$

$$+ \frac{1}{E^2}\left[\frac{T_4T_{11}}{4\pi} - m\,\frac{T_{12}}{4\pi}\,\frac{T_{11}}{2\pi}\right]\frac{b\dot{\beta}}{V} + \frac{1}{E^2}\left[\frac{T_3}{2\pi}\right]\frac{b^2\ddot{\beta}}{V^2} \tag{23}$$

All values of T are a function of a or c only with the exception of T_{13} and T_9.

$$T_{13} = -\frac{1}{2}\left(T_7 + (c-a)\,T_1\right)$$

From reference 3 the value of T_{17} is

$$T_{17} = -2T_9 - T_1 + \left(a - \frac{1}{2}\right)T_4 = -\frac{1}{3}\left(1-c^2\right)^{3/2} - T_1 - \frac{T_4}{2}$$

For the purposes of this report the following coefficients defined in terms of the notation of reference 1 will be adopted:

$$
\left.
\begin{aligned}
K_{\alpha'} &= \left(\frac{1}{2} - a\right) & C_{m_A} &= \frac{1}{2}\left(a + \frac{1}{2}\right) \\
K_\beta &= T_{10}/\pi & C_{H_A} &= -T_{12}/4\pi E^2 \\
K_{\dot\beta} &= T_{11}/2\pi & E &= C_f/C_T \\
C_{L_{B\dot\alpha'}} &= \pi & C_{m_{B\dot\alpha'}} &= -\frac{\pi}{2}\left(\frac{1}{2} - a\right) \\
C_{L_{B\ddot\alpha'}} &= -a\pi & C_{m_{B\ddot\alpha'}} &= -\frac{\pi}{2}\left(\frac{1}{8} + a^2\right) \\
C_{L_{B\ddot h}} &= \pi & C_{m_{B\ddot h}} &= \frac{\pi}{2} a \\
C_{L_{B\beta}} &= 0 & C_{m_{B\beta}} &= -(T_4 + T_{10})/2 \\
C_{L_{B\dot\beta}} &= -T_4 & C_{m_{B\dot\beta}} &= -\left(T_1 - T_8 + \frac{T_{11}}{2}\right)/2 \\
C_{L_{B\ddot\beta}} &= -T_1 & C_{m_{B\ddot\beta}} &= \frac{T_7}{2} \\
C_{H_{B\dot\alpha'}} &= -T_{17}/2E^2 & C_{H_{B\beta}} &= -(T_5 - T_4T_{10})/2\pi E^2 \\
C_{H_{B\ddot\alpha'}} &= T_7/2E^2 & C_{H_{B\dot\beta}} &= T_4T_{11}/4\pi E^2 \\
C_{H_{B\ddot h}} &= T_1/2E^2 & C_{H_{B\ddot\beta}} &= T_3/2\pi E^2
\end{aligned}
\right\} \quad (24)
$$

Values of these coefficients have been calculated and plotted in figures 2, 3, and 8 to 13. Then

$$
C_L = m\alpha' + \left[C_{L_{B\dot\alpha'}} + mK_{\dot\alpha'}\right]\frac{b\dot\alpha'}{v} + \left[C_{L_{B\ddot\alpha'}}\right]\frac{b^2\ddot\alpha'}{v^2} + m\frac{\dot h}{v} + \left[C_{L_{B\ddot h}}\right]\frac{b\ddot h}{v^2}
$$

$$
+ \left[mK_\beta\right]\beta + \left[C_{L_{B\dot\beta}} + mK_{\dot\beta}\right]\frac{b\dot\beta}{v} + \left[C_{L_{B\ddot\beta}}\right]\frac{b^2\ddot\beta}{v^2} \qquad (25)
$$

$$C_m = \left[C_{m_A}\ m\right]\alpha' + \left[C_{m_{B\dot{\alpha}'}} + mK_{\dot{\alpha}'}\ C_{m_A}\right]\frac{b\dot{\alpha}'}{V} + \left[C_{m_{B\ddot{\alpha}'}}\right]\frac{b^2\ddot{\alpha}'}{V^2}$$

$$+ \left[C_{m_A}\ m\right]\frac{\dot{h}}{V} + \left[C_{m_{B\ddot{h}}}\right]\frac{b\ddot{h}}{V^2} + \left[C_{m_{B\beta}} + mK_\beta\ C_{m_A}\right]\beta$$

$$+ \left[C_{m_{B\dot{\beta}}} + mK_{\dot{\beta}}\ C_{m_A} - \frac{(c-a)}{2}\ C_{L_{B\dot{\beta}}}\right]\frac{b\dot{\beta}}{V}$$

$$+ \left[C_{m_{B\ddot{\beta}}} - \frac{(c-a)}{2}\ C_{L_{B\ddot{\beta}}}\right]\frac{b^2\ddot{\beta}}{V^2} \qquad (26)$$

$$C_H = \left[m\ C_{H_A}\right]\alpha' + \left[C_{H_{B\dot{\alpha}'}} + m\ K_{\dot{\alpha}'}\ C_{H_A}\right]\frac{b\dot{\alpha}'}{V}$$

$$+ \left[C_{H_{B\ddot{\alpha}'}} - \frac{(c-a)}{2}\ C_{L_{B\ddot{\beta}}}\right]\frac{b^2\ddot{\alpha}'}{V^2} + \left[m\ C_{H_A}\right]\frac{\dot{h}}{V} + \left[C_{H_{B\ddot{h}}}\right]\frac{b\ddot{h}}{V^2}$$

$$+ \left[C_{H_{B\beta}} + m\ K_\beta\ C_{H_A}\right]\beta + \left[C_{H_{B\dot{\beta}}} + m\ K_{\dot{\beta}}\ C_{H_A}\right]\frac{b\dot{\beta}}{V} + \left[C_{H_{B\ddot{\beta}}}\right]\frac{b^2\ddot{\beta}}{V^2} \qquad (27)$$

SUMMARY OF STABILITY AND PRESSURE COEFFICIENTS

Following is a list of equations for calculating the pressure and stability derivatives. In some cases equations involving δ_t are not written, since the tab effects can be calculated from the equations for δ by use of the proper chord ratios.

I. Pressure Coefficients

$$\left.\begin{aligned}
P_{\alpha} &= m\ P_A \\
P_{D(\alpha)} &= \left[P_{B\ddot{h}}\right]\left(\frac{C_T}{M.A.C.}\right) \\
P_{D(\theta)} &= \left[P_{B\dot{\alpha}'} - P_{B\ddot{h}} + m\ K_{\dot{\alpha}'}\ P_A\right]\left(\frac{C_T}{M.A.C.}\right) \\
P_{D^2(\theta)} &= \left[P_{B\ddot{\alpha}'} - a\ P_{B\ddot{h}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
P_{\delta} &= \left[P_{B\beta} + m\ K_{\beta}\ P_A\right] \\
P_{D(\delta)} &= \left[P_{B\dot{\beta}} + m\ K_{\dot{\beta}}\ P_A\right]\left(\frac{C_T}{M.A.C.}\right) \\
P_{D^2(\delta)} &= \left[P_{B\ddot{\beta}}\right]\left(\frac{C_T}{M.A.C.}\right)^2
\end{aligned}\right\} \quad (28)$$

II. Airfoil Lift Coefficients

$$\left.\begin{aligned}
C_{L\alpha} &= m \\
C_{L_{D(\alpha)}} &= \left[C_{L_{B\ddot{h}}}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{L_{D(\theta)}} &= \left[C_{L_{B\dot{\alpha}'}} - C_{L_{B\ddot{h}}} + m\ K_{\dot{\alpha}'}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{L_{D^2(\theta)}} &= \left[C_{L_{B\ddot{\alpha}'}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
C_{L_{\delta}} &= \left[m\ K_{\beta}\right] \\
C_{L_{D(\delta)}} &= \left[C_{L_{B\dot{\beta}}} + m\ K_{\dot{\beta}}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{L_{D^2(\delta)}} &= \left[C_{L_{B\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2
\end{aligned}\right\} \quad (29)$$

III. Airfoil Pitching Moment Coefficients

$$\left.\begin{aligned}
C_{m_\alpha} &= \left[m\ C_{m_A}\right] \\
C_{m_{D(\alpha)}} &= \left[C_{m_{B\ddot{h}}}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{m_{D(\theta)}} &= \left[C_{m_{B\dot{\alpha}'}} - C_{m_{B\ddot{h}}} + m\ K_{\dot{\alpha}'}\ C_{m_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{m_{D^2(\theta)}} &= \left[C_{m_{B\ddot{\alpha}'}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
C_{m_\delta} &= \left[C_{m_{B\beta}} + m\ K_\beta\ C_{m_A}\right] \\
C_{m_{D(\delta)}} &= \left[C_{m_{B\dot{\beta}}} - (1-E-H)\ C_{L_{B\dot{\beta}}} + m\ K_{\dot{\beta}}\ C_{m_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{m_{D^2(\delta)}} &= \left[C_{m_{B\ddot{\beta}}} - (1-E-H)\ C_{L_{B\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2
\end{aligned}\right\} \quad (30)$$

IV. Flap Hinge Moment Coefficients

$$\left.\begin{aligned}
C_{H_{f\alpha}} &= \left[m\ C_{H_A}\right] \\
C_{H_{fD(\alpha)}} &= \left[C_{H_{B\ddot{h}}}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{fD(\theta)}} &= \left[C_{H_{B\dot{\alpha}'}} - C_{H_{B\ddot{h}}} + m\ K_{\dot{\alpha}'}\ C_{H_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{fD^2(\theta)}} &= \left[C_{H_{B\ddot{\alpha}'}} - \frac{(1-E-H)}{E^2}\ C_{L_{B\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
C_{H_{f\delta}} &= \left[C_{H_{B\beta}} + m\ K_\beta\ C_{H_A}\right] \\
C_{H_{fD(\delta)}} &= \left[C_{H_{B\dot{\beta}}} + m\ K_{\dot{\beta}}\ C_{H_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{fD^2(\delta)}} &= \left[C_{H_{B\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
C_{H_{f\delta_t}} &= C_{H_{B\delta_t}} + m\ K_{\delta_t}\ C_{H_A} \\
C_{H_{fD(\delta_t)}} &= \left[C_{H_{B\dot{\delta}_t}} + m\ K_{\dot{\delta}_t}\ C_{H_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{fD^2(\delta_t)}} &= \left[C_{H_{B\ddot{\delta}_t}}\right]\left(\frac{C_T}{M.A.C.}\right)^2
\end{aligned}\right\} \quad (31)$$

V. Tab Hinge Moment Coefficients

$$
\left.
\begin{aligned}
C_{H_{t\alpha}} &= m\ C_{H_A} \\
C_{H_{tD(\alpha)}} &= \left[C_{H_{B\dot{\alpha}}} + m\ K_{\dot{\alpha}}\ C_{H_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{t\delta_t}} &= C_{H_{B\beta}} + m\ K_\beta\ C_{H_A} \\
C_{H_{tD(\delta_t)}} &= \left[C_{H_{B\dot{\beta}}} + m\ K_{\dot{\beta}}\ C_{H_A}\right]\left(\frac{C_T}{M.A.C.}\right) \\
C_{H_{tD^2(\delta_t)}} &= \left[C_{H_{B\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2 \\
C_{H_{t\delta}} &= C_{H_{tB\beta}} + m\ K_\beta\ C_{H_A} \\
C_{H_{tD(\delta)}} &= \left[C_{H_{tB\dot{\beta}}} + m\ K_{\dot{\beta}}\ C_{H_A}\right]\left(\frac{C_{\dot{T}}}{M.A.C.}\right) \\
C_{H_{tD^2(\delta)}} &= \left[C_{H_{tB\ddot{\beta}}}\right]\left(\frac{C_T}{M.A.C.}\right)^2
\end{aligned}
\right\} \qquad (32)
$$

The basic tab hinge moment derivatives $C_{H_{tB\beta}}$, $C_{H_{tB\dot{\beta}}}$, and $C_{H_{tB\ddot{\beta}}}$ were obtained from a graphical calculation of the first moments about the tab hinge axis of the areas under the curves of $P_{B\beta}$, $P_{B\dot{\beta}}$, and $P_{B\ddot{\beta}}$. These curves are presented in figures 14, 15, and 16.

Curves of $C_{H_{B\delta_t}}$ which are shown in figure 17 are taken from reference 2, and the curves of $C_{H_{B\dot{\delta}_t}}$ and $C_{H_{B\ddot{\delta}_t}}$, figures 18 and 19, were constructed from a graphical calculation of the first moments about the flap hinge axis of the areas under the curves of $P_{B\dot{\beta}}$ and $P_{B\ddot{\beta}}$ for the deflected tab.

Curtiss-Wright Corporation
Lambert Field, Saint Louis, Mo., April 11, 1944.

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2. Ames, Milton B., Jr., and Sears, Richard I.: Determination of Control-Surface Characteristics from NACA Plain-Flap and Tab Data. NACA Rep. No. 721, 1941.

3. Theodorsen, Theodore, and Garrick, I. E.: Mechanism of Flutter - A Theoretical and Experimental Investigation of the Flutter Problem. NACA Rep. No. 685, 1940.

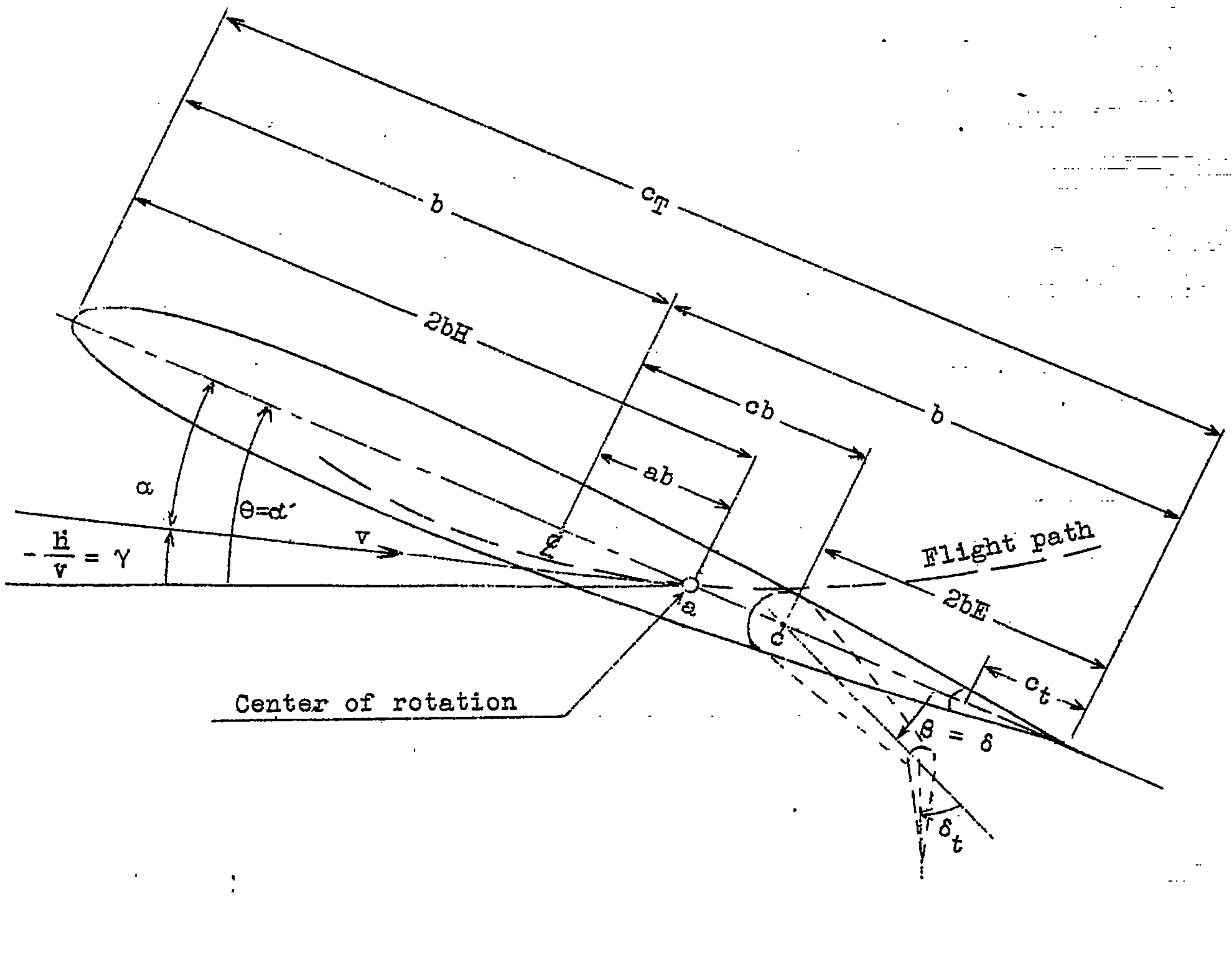

Figure 1.- Airfoil geometry.

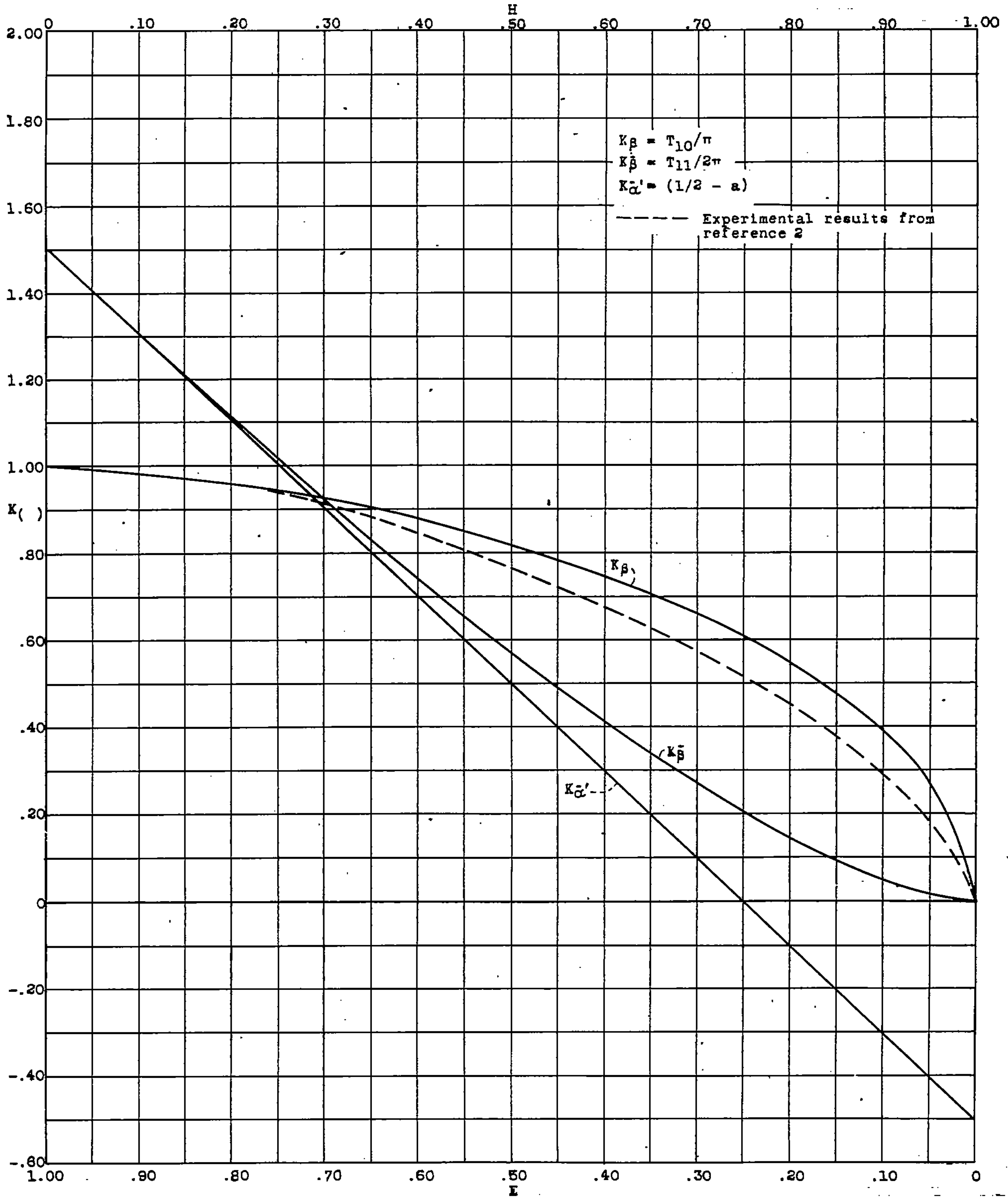

Figure 2.- Airfoil effectiveness.

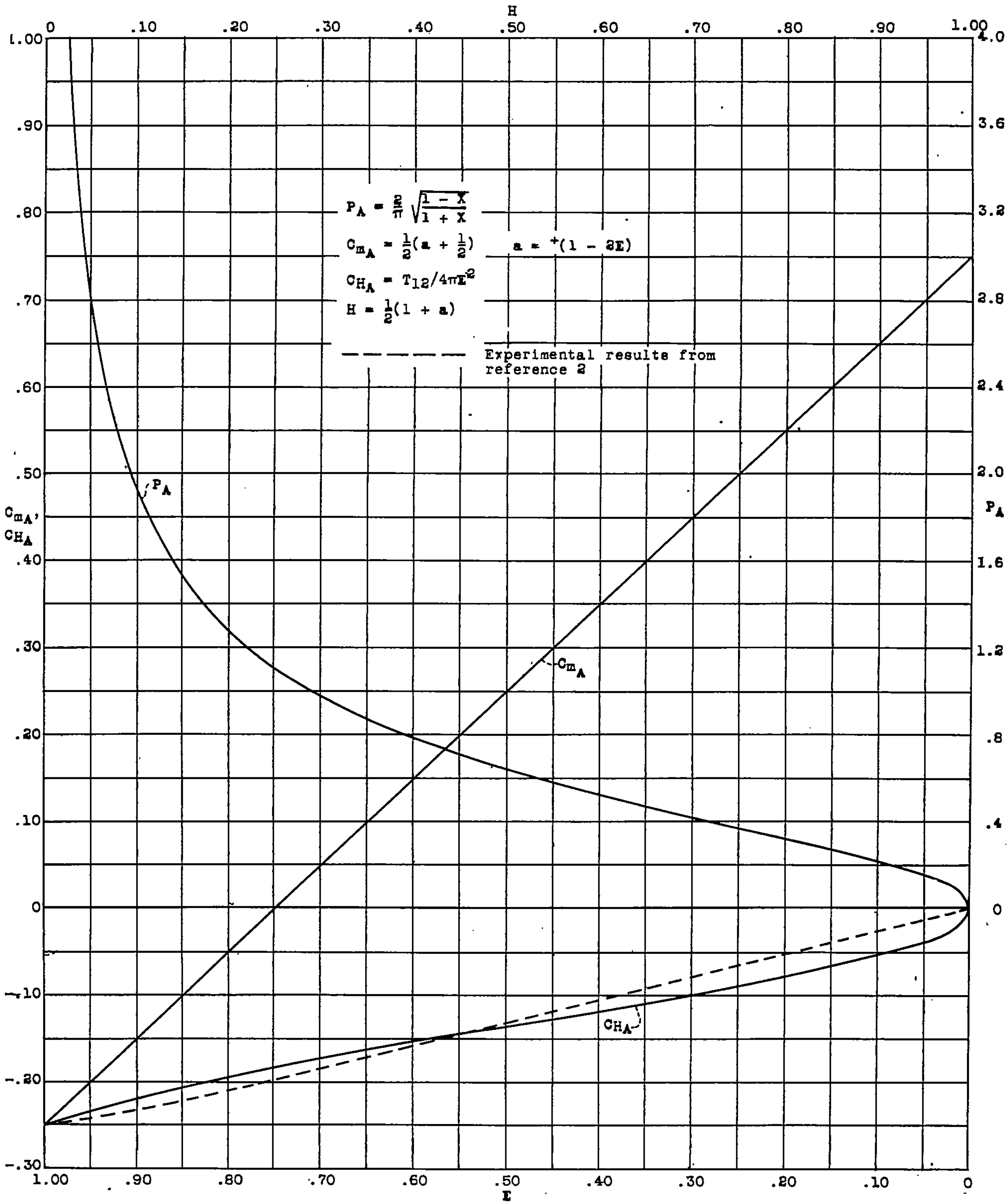

Figure 3.- Additional force coefficients.

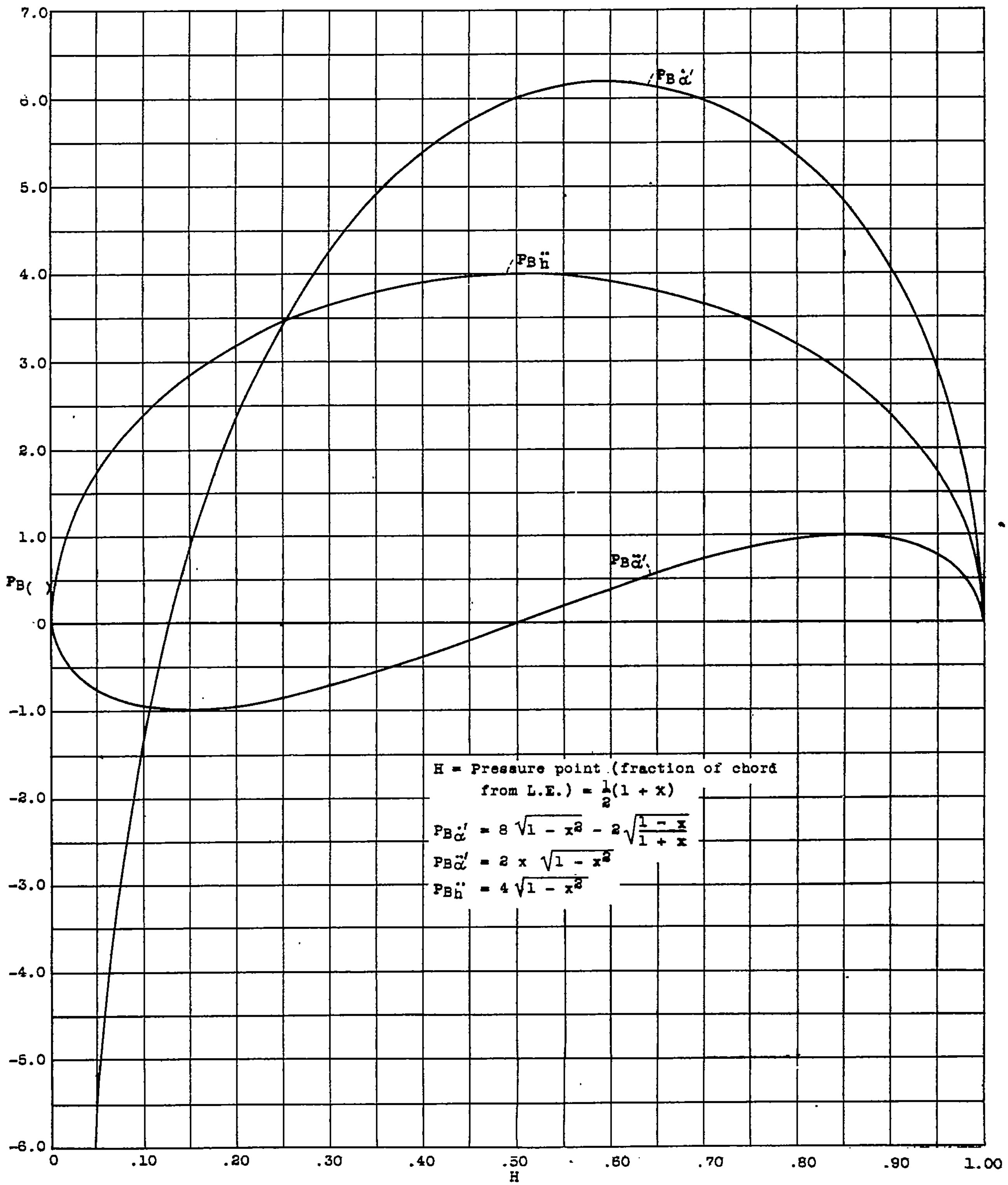

Figure 4.- Basic pressure distributions for angle of attack changes.

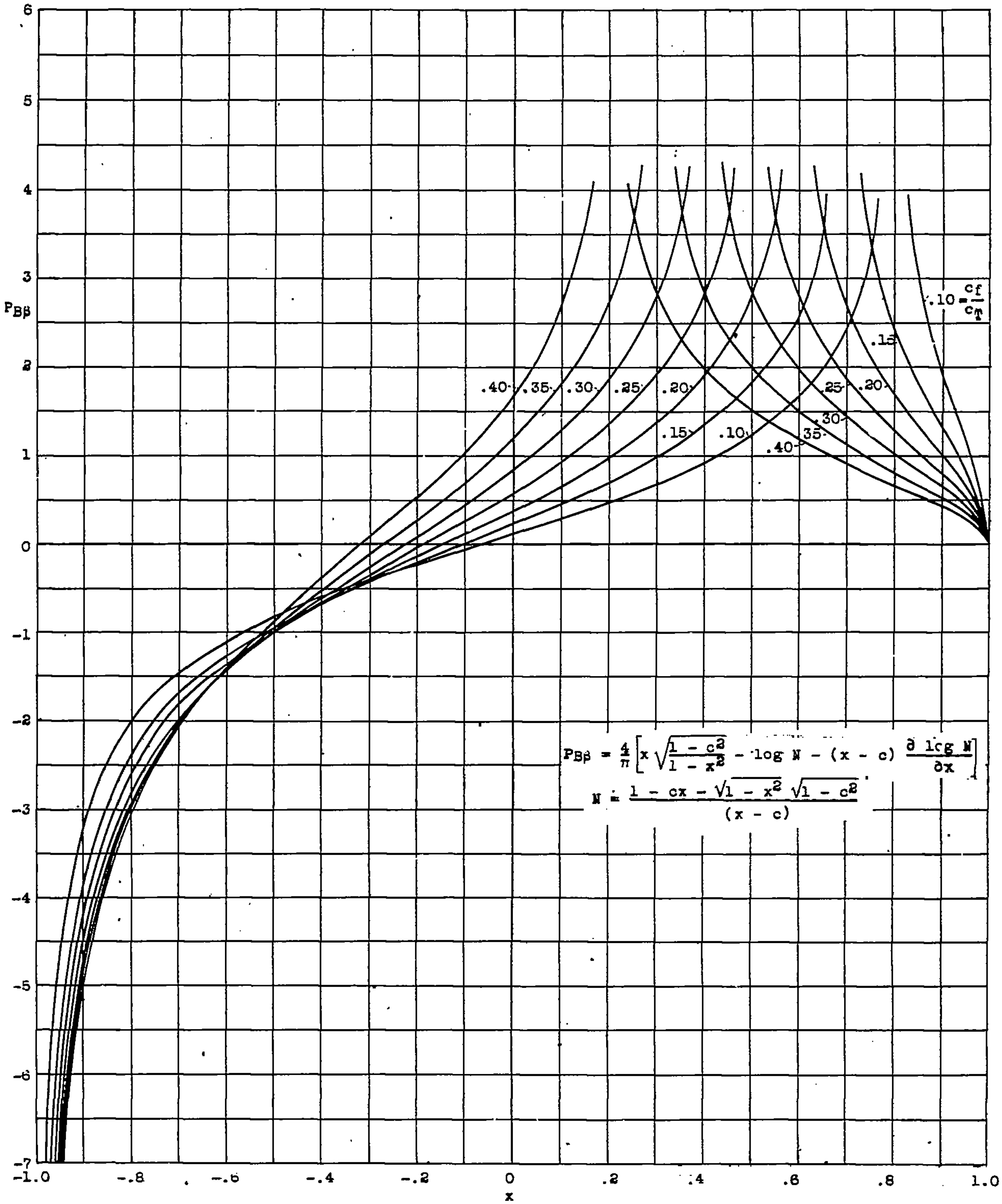

Figure 5.- Basic pressure distribution $P_{B\beta}$ (10, 15, 20, 25, 30, 35, and 40 percent chord flaps).

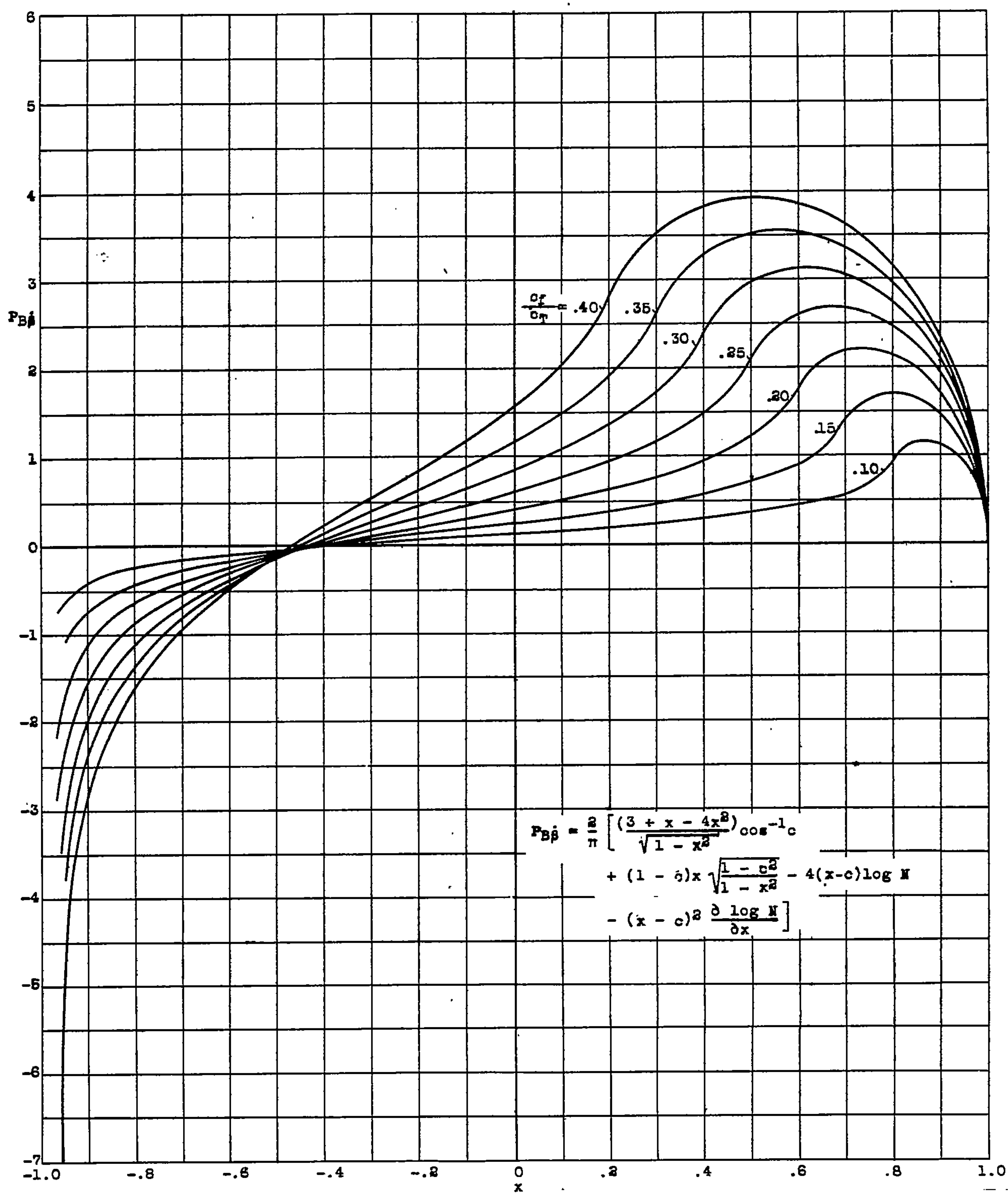

Figure 6.- Basic pressure distribution $P_{B\dot{\beta}}$ (10, 15, 20, 25, 30, 35, and 40 percent chord flaps).

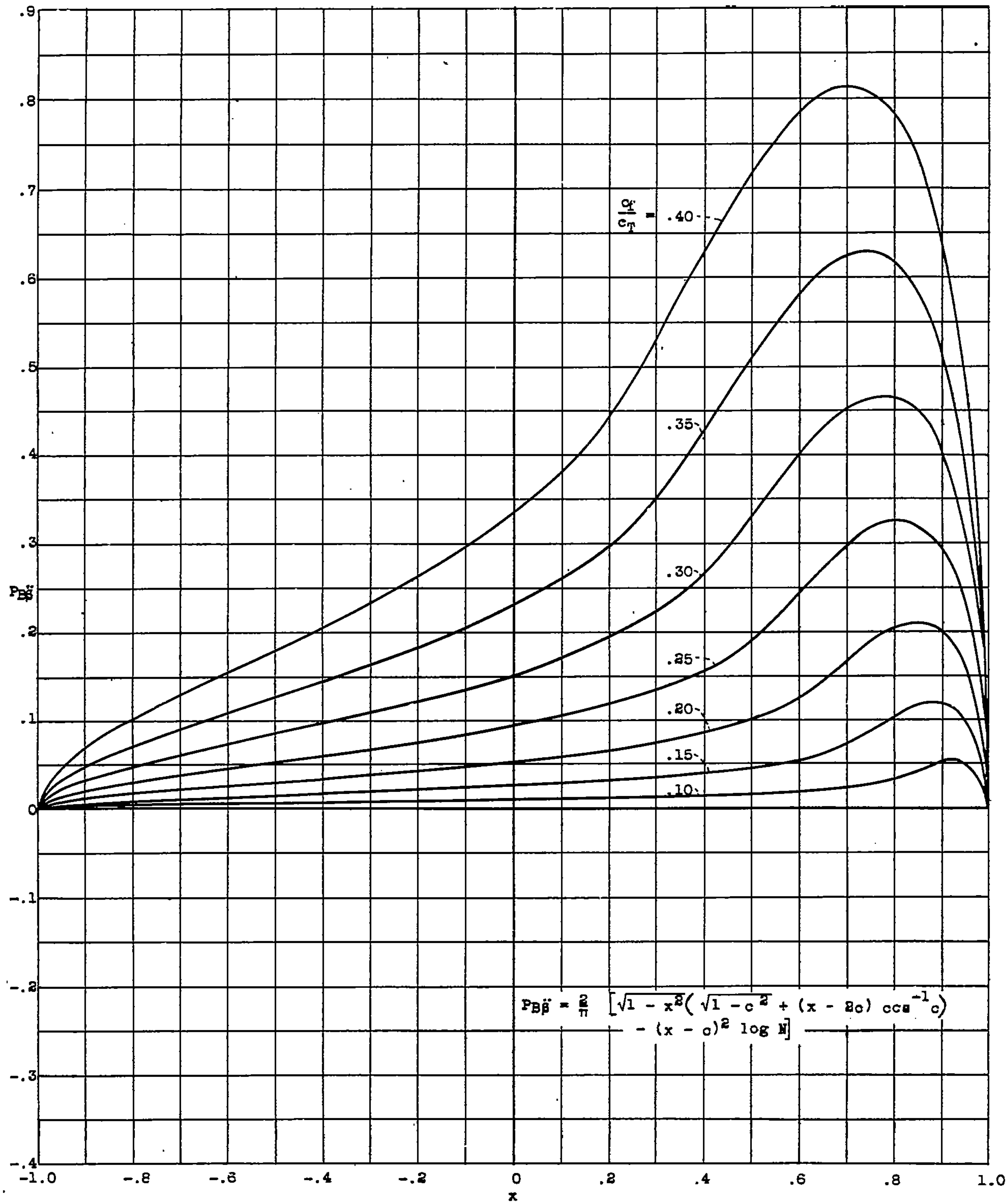

Figure 7.- Basic pressure distribution $P_{B\ddot{\beta}}$ (10, 15, 20, 25, 30, 35, and 40 percent chord flaps).

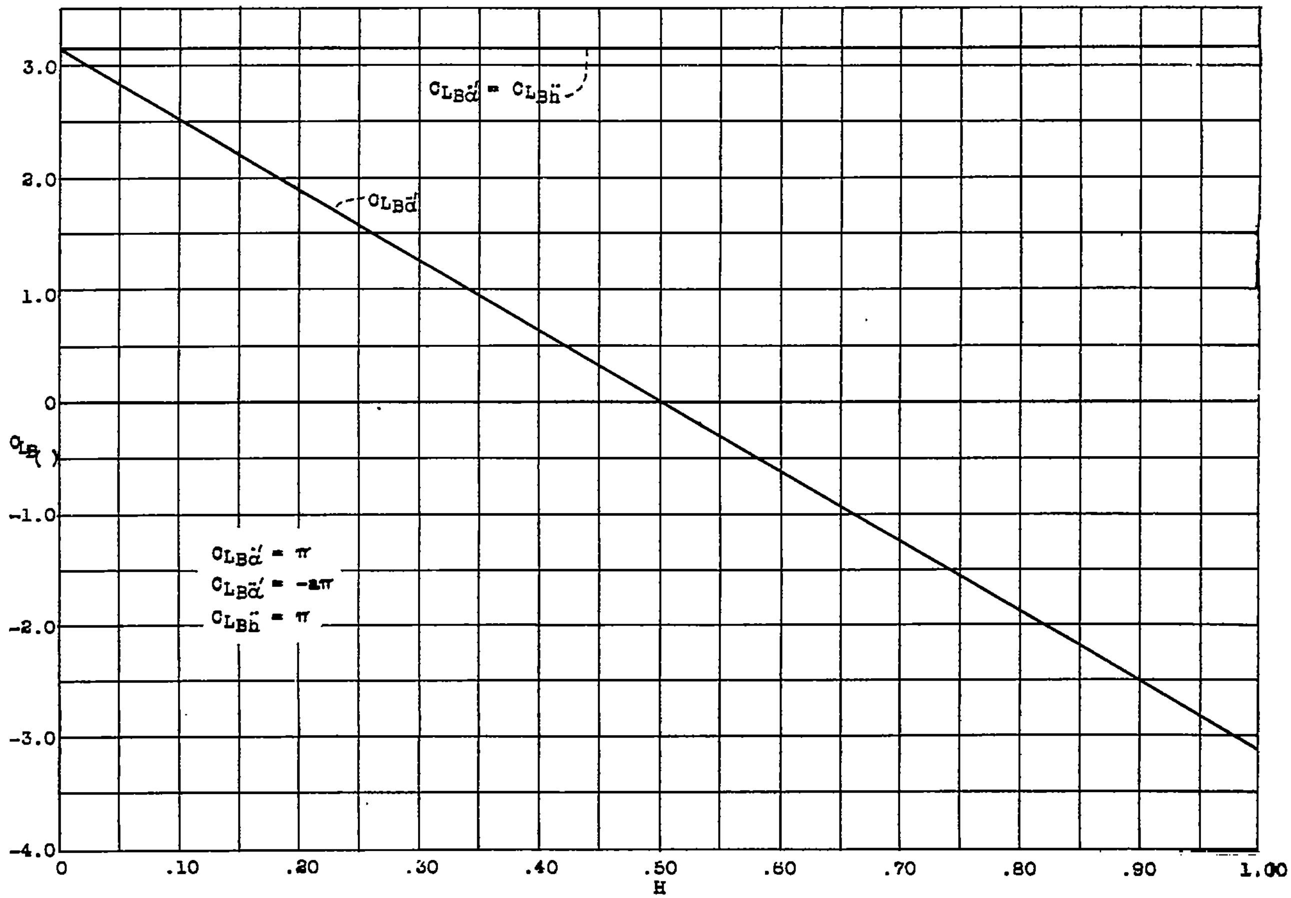

Figure 8.- Basic lift coefficients for changes in angle of attack.

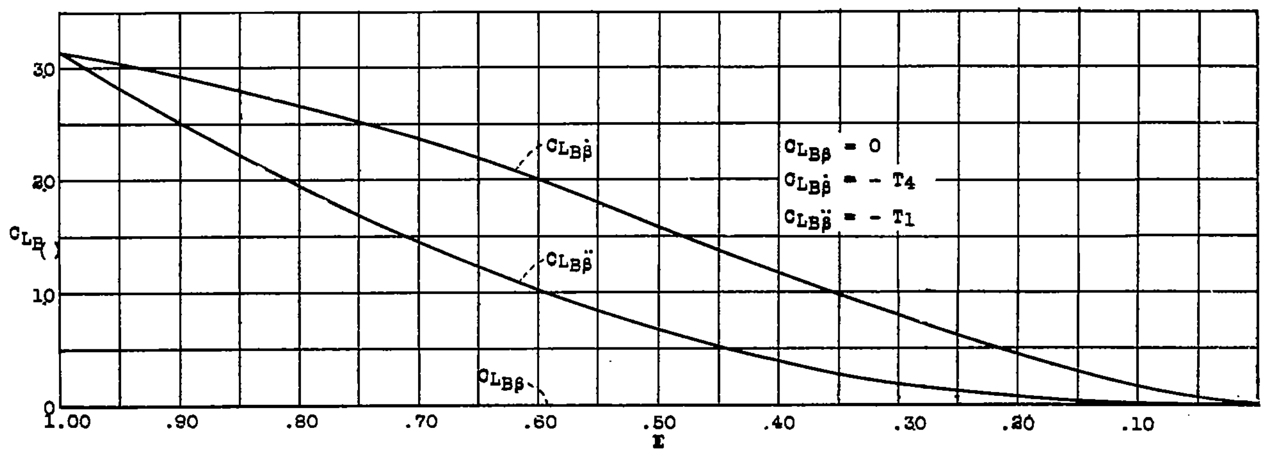

Figure 9.- Basic lift coefficients for elevator changes.

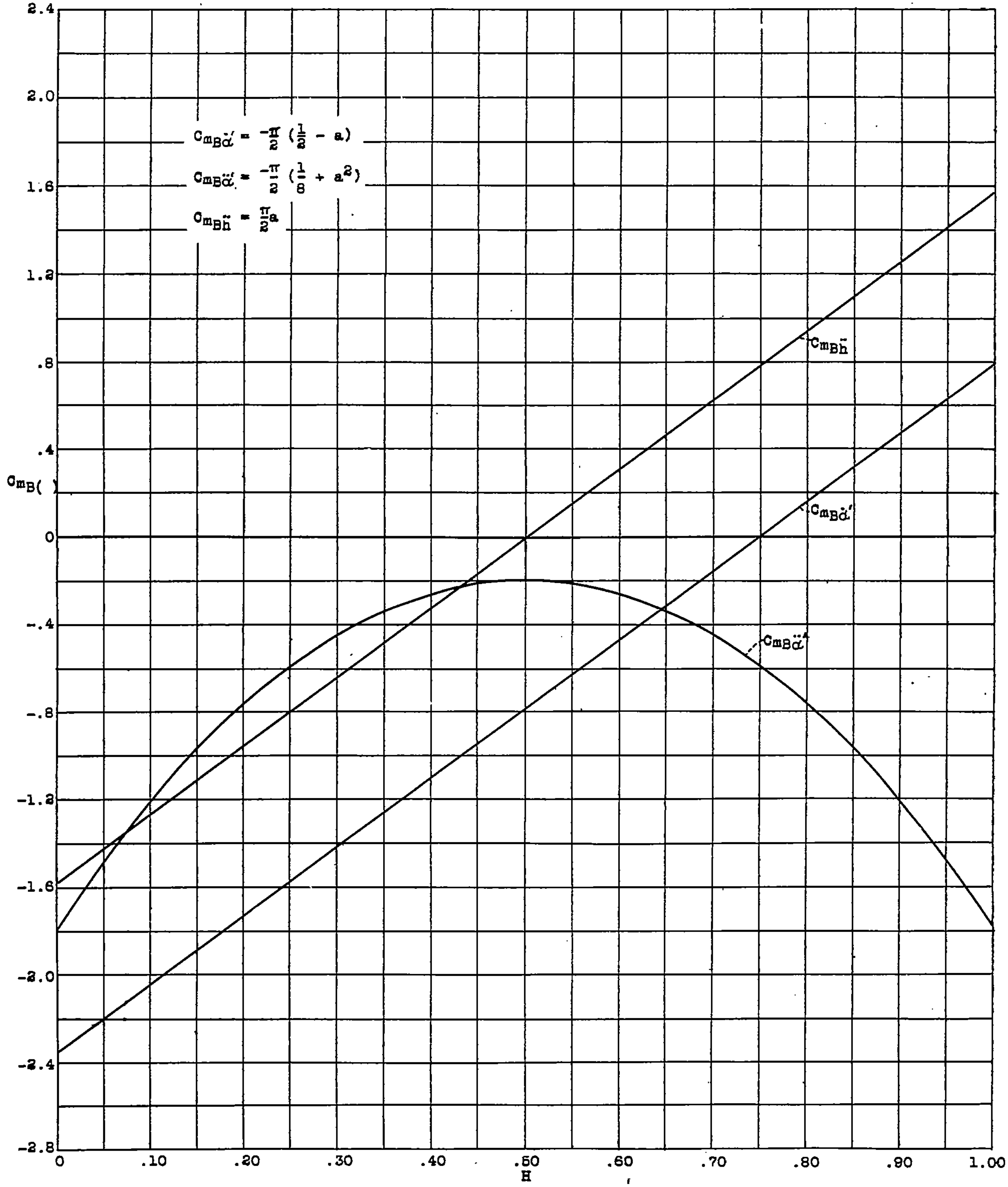

Figure 10.- Basic pitching moment coefficients for angle of attack changes.

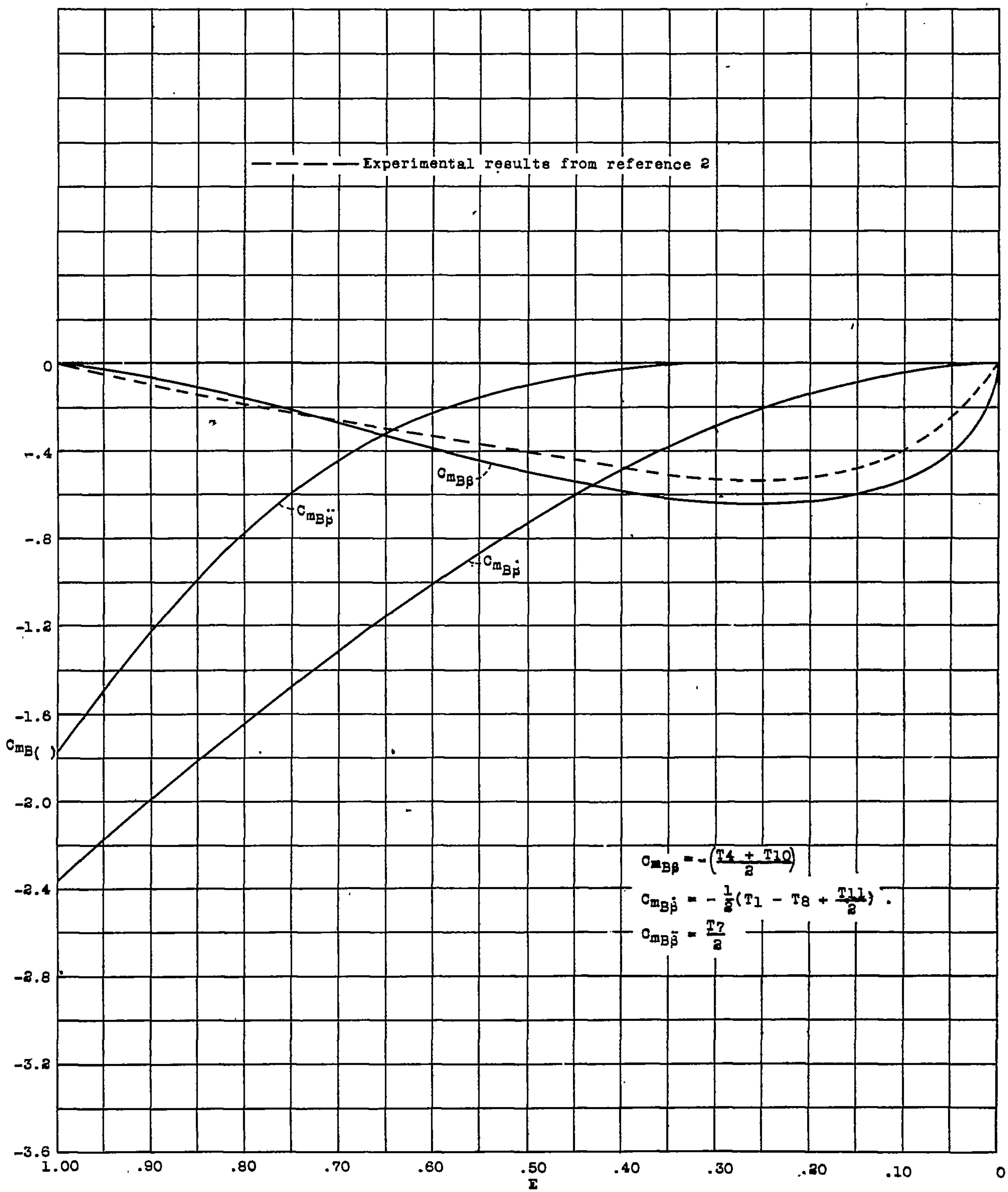

Figure 11.- Basic pitching moment coefficients for elevator angle changes.

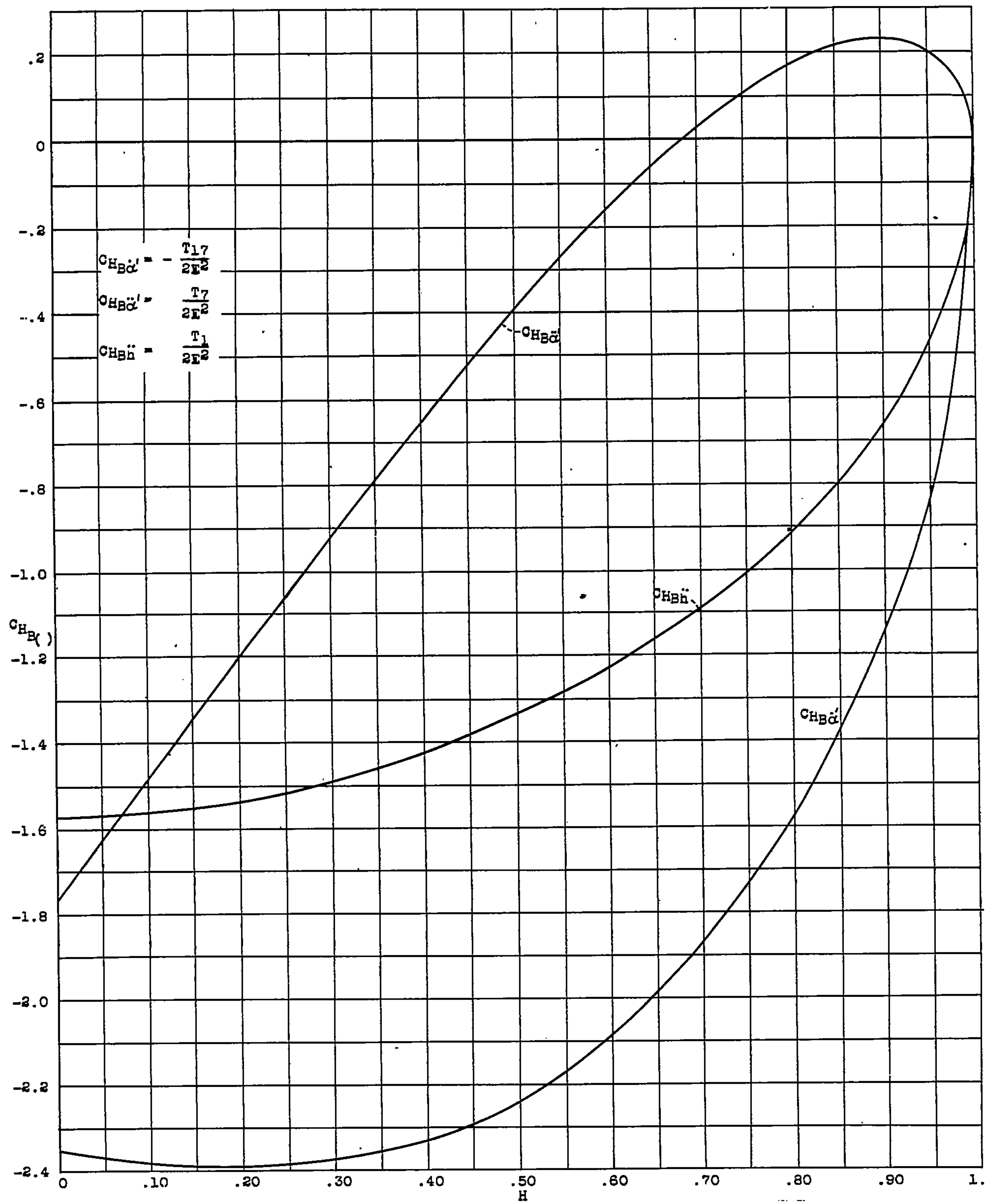

Figure 12.- Basic hinge moment coefficients for changes in angle of attack.

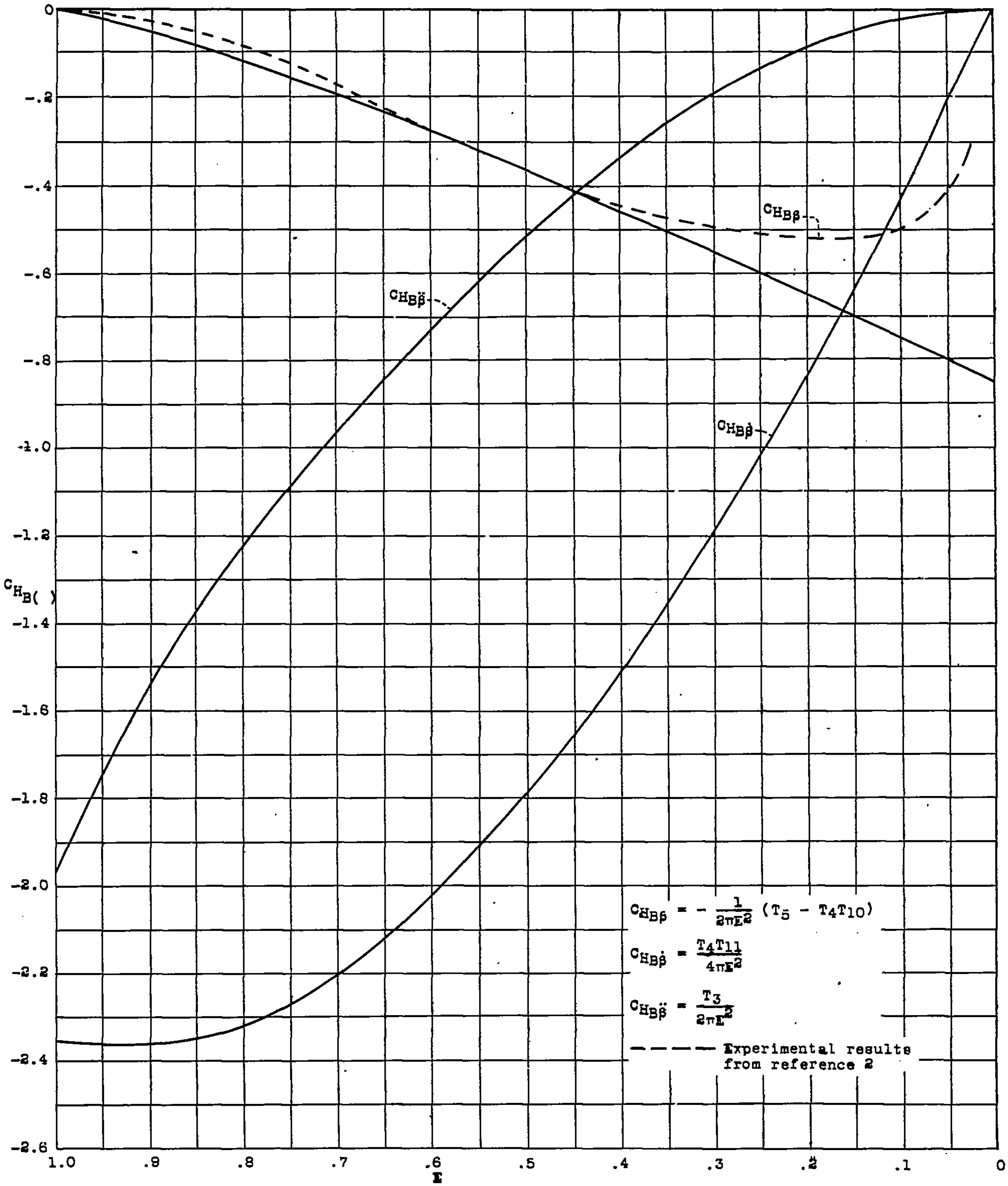

Figure 13.- Basic hinge moment coefficients for elevator changes.

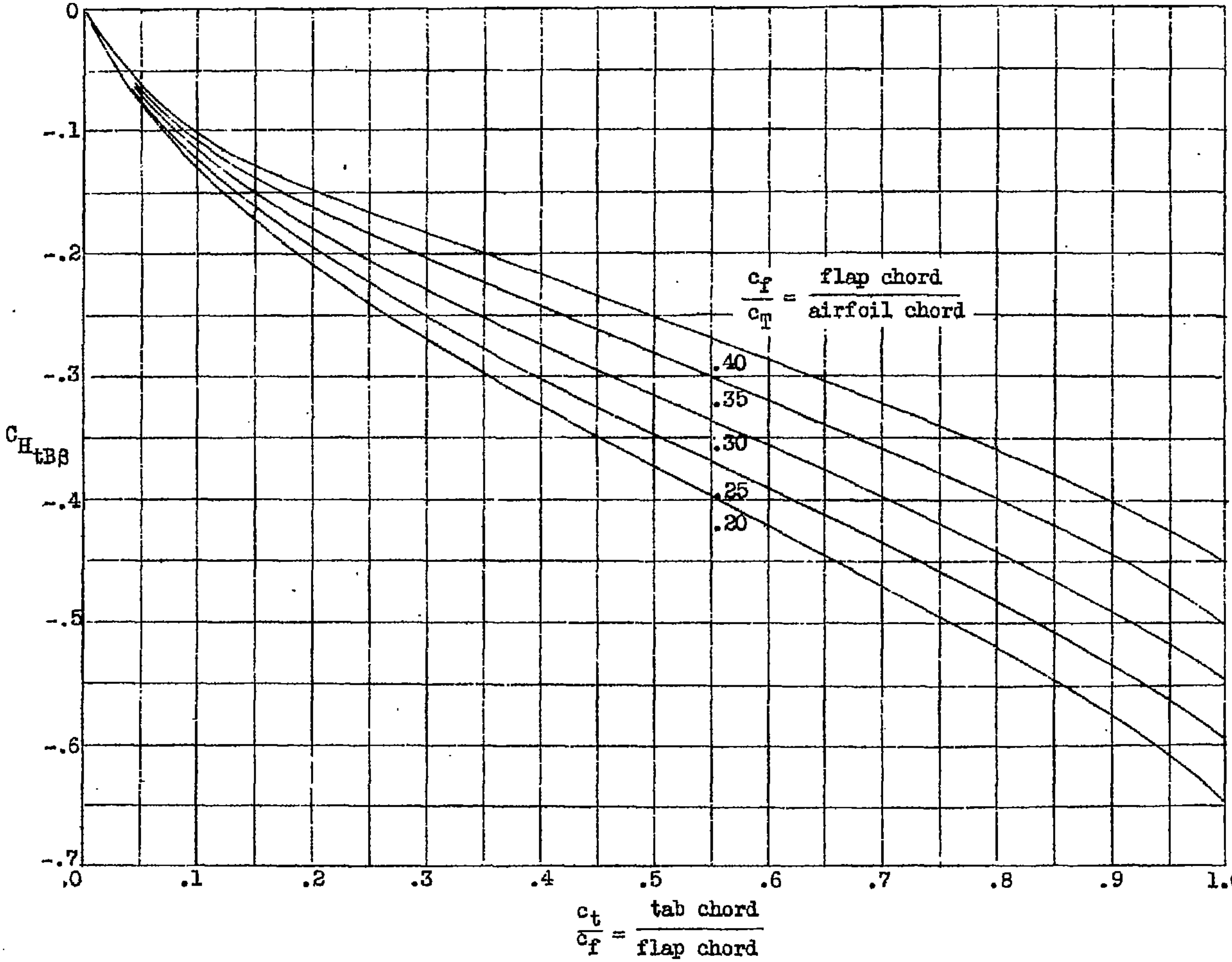

Figure 14.- Basic tab hinge moment derivative, $C_{H_{tB\beta}}$, for elevator changes (per radian).

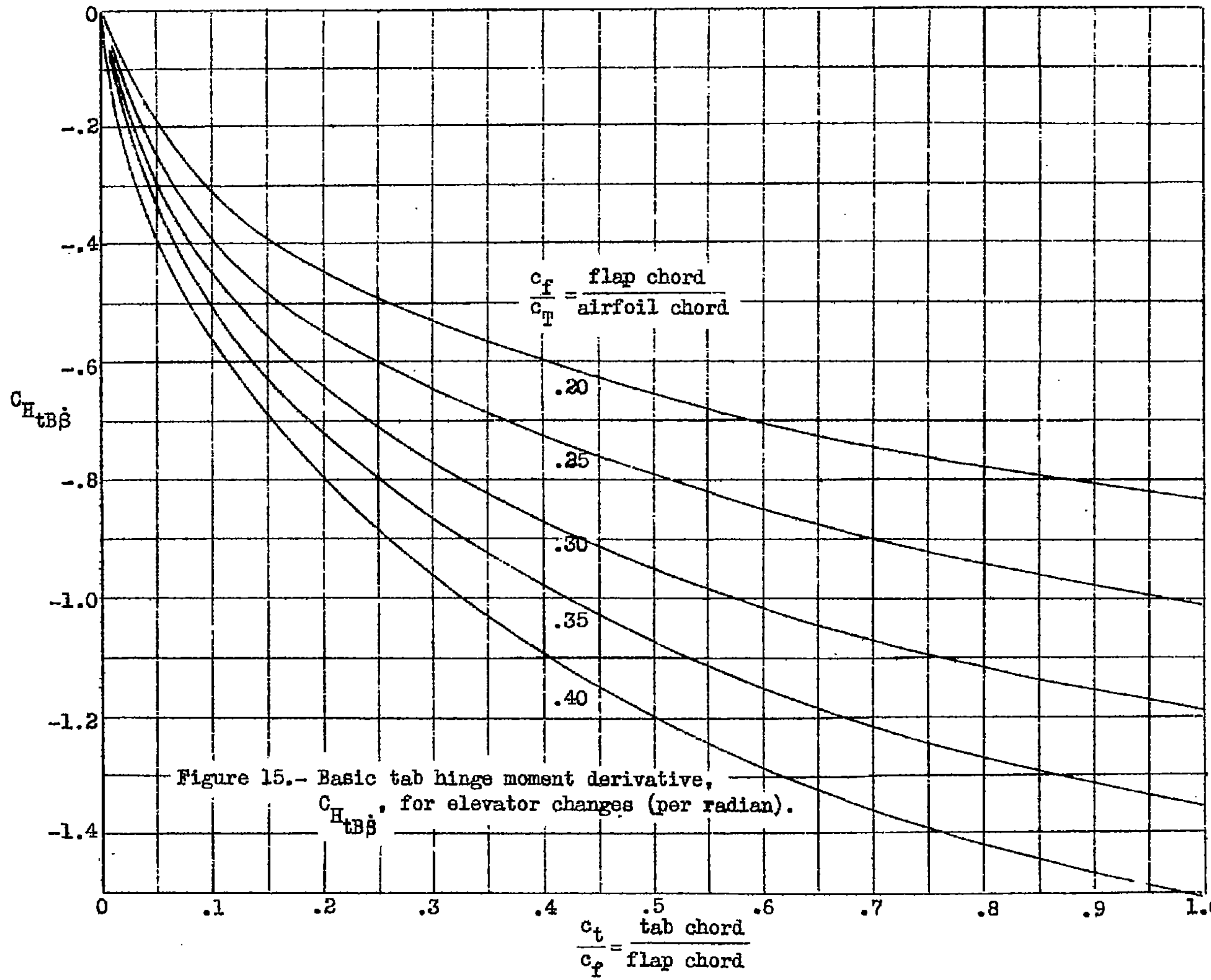

Figure 15.- Basic tab hinge moment derivative, $C_{H_{tB\dot{\beta}}}$, for elevator changes (per radian).

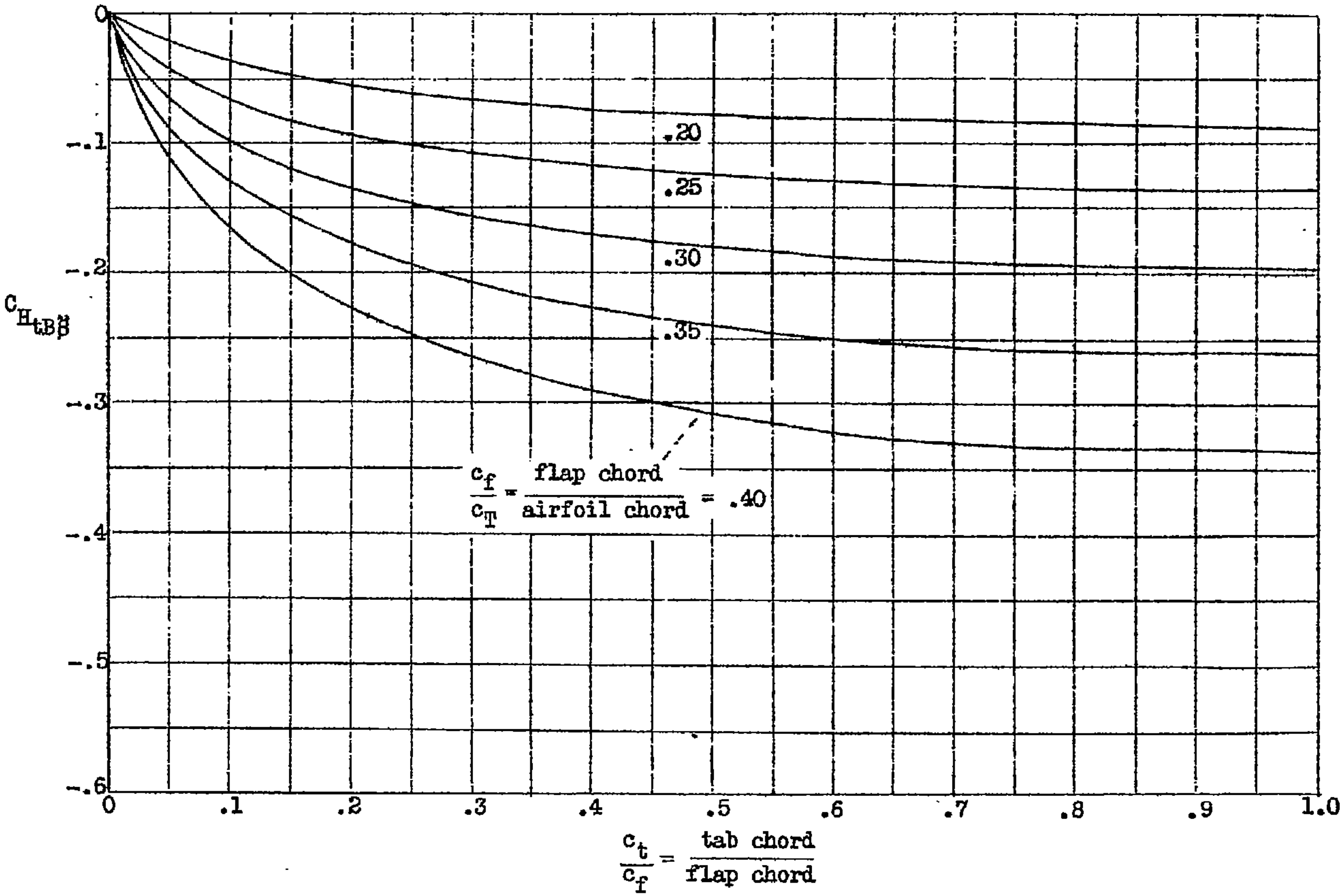

Figure 16.- Basic tab hinge moment derivative, $C_{H_{tB\beta}}$, for elevator changes (per radian).

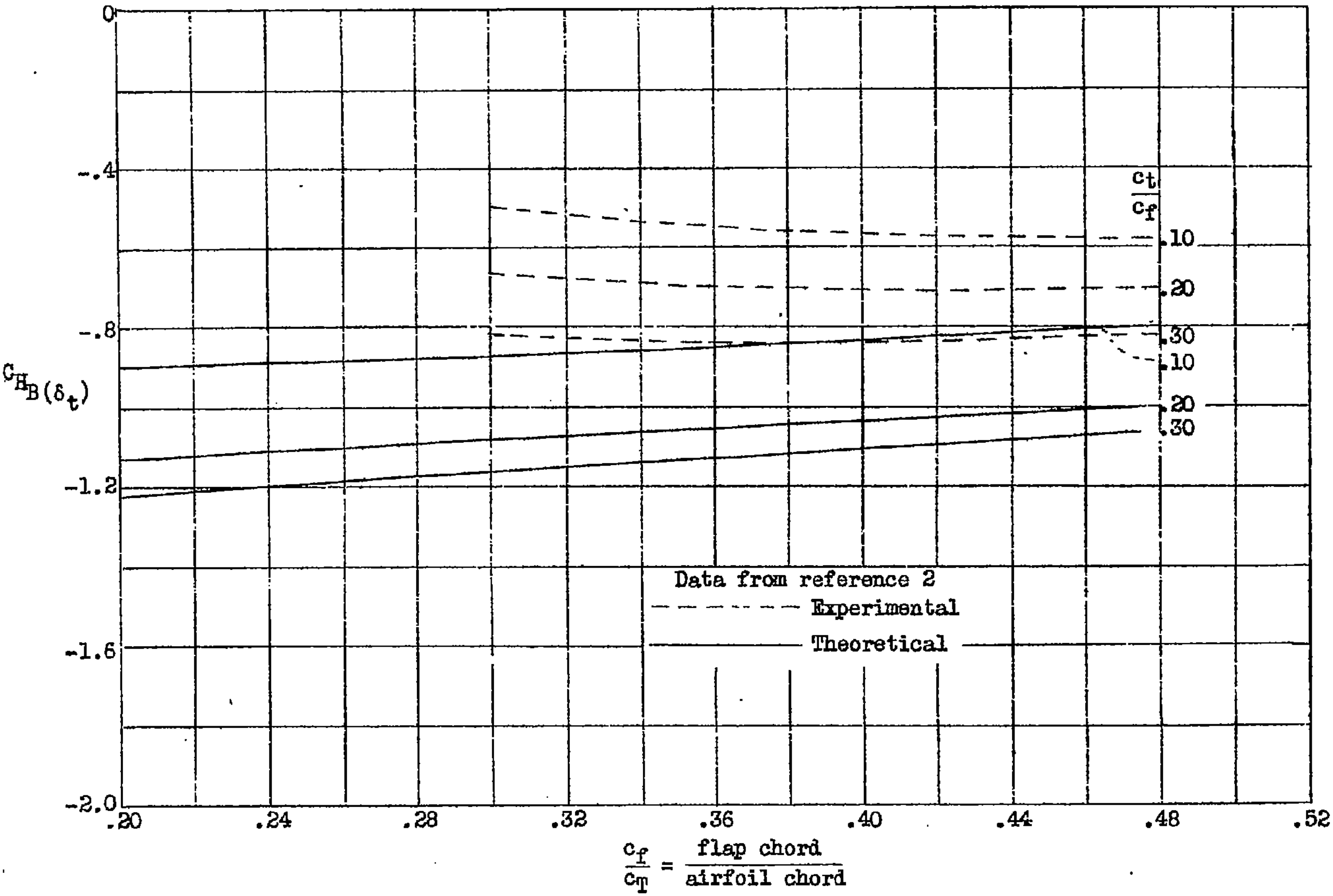

Figure 17.- Basic elevator hinge moment derivative, $C_{H_B(\delta_t)}$ for tab changes (per radian).

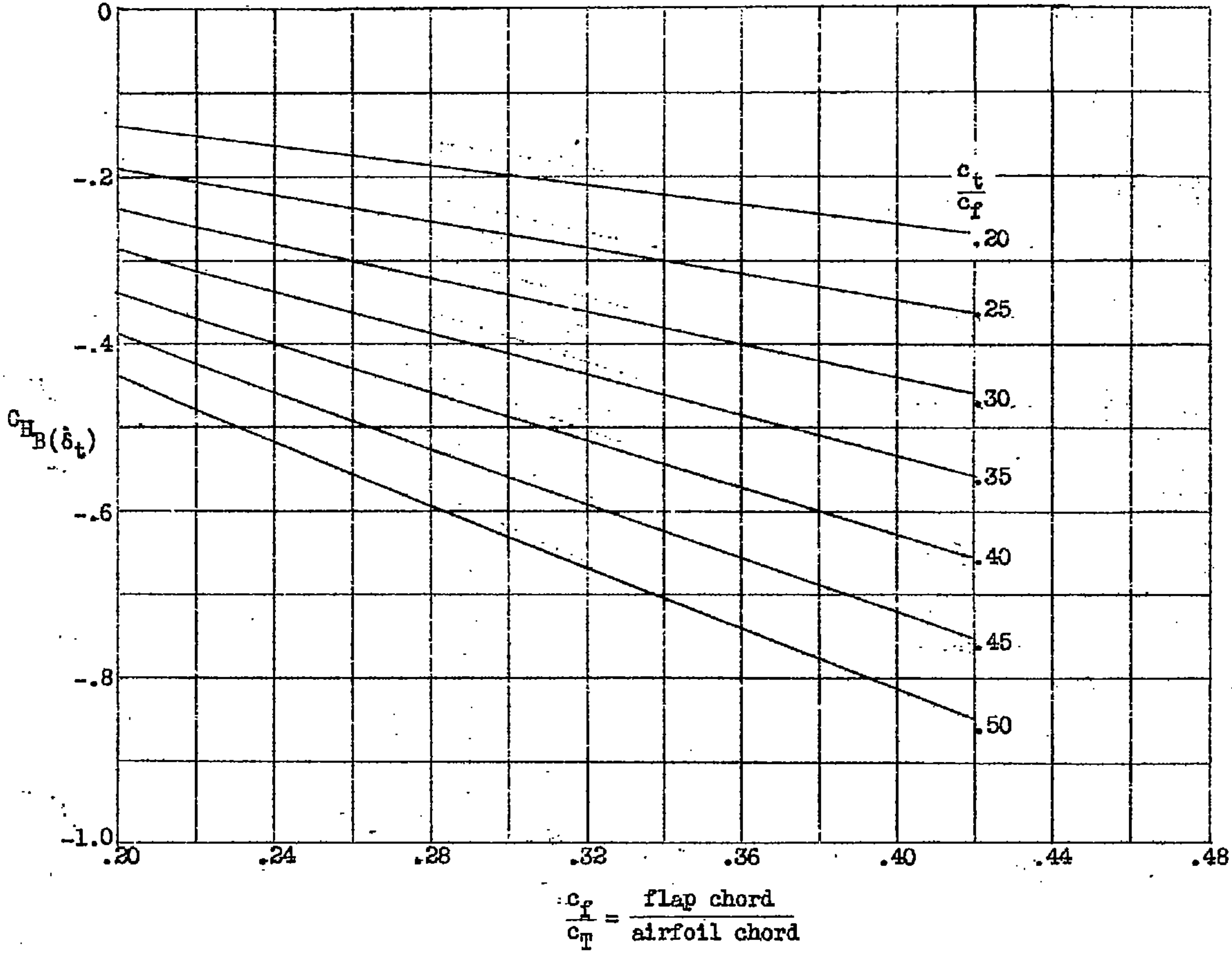

Figure 18.- Basic elevator hinge moment derivative, $C_{H_B(\delta_t)}$ for tab changes (per radian).

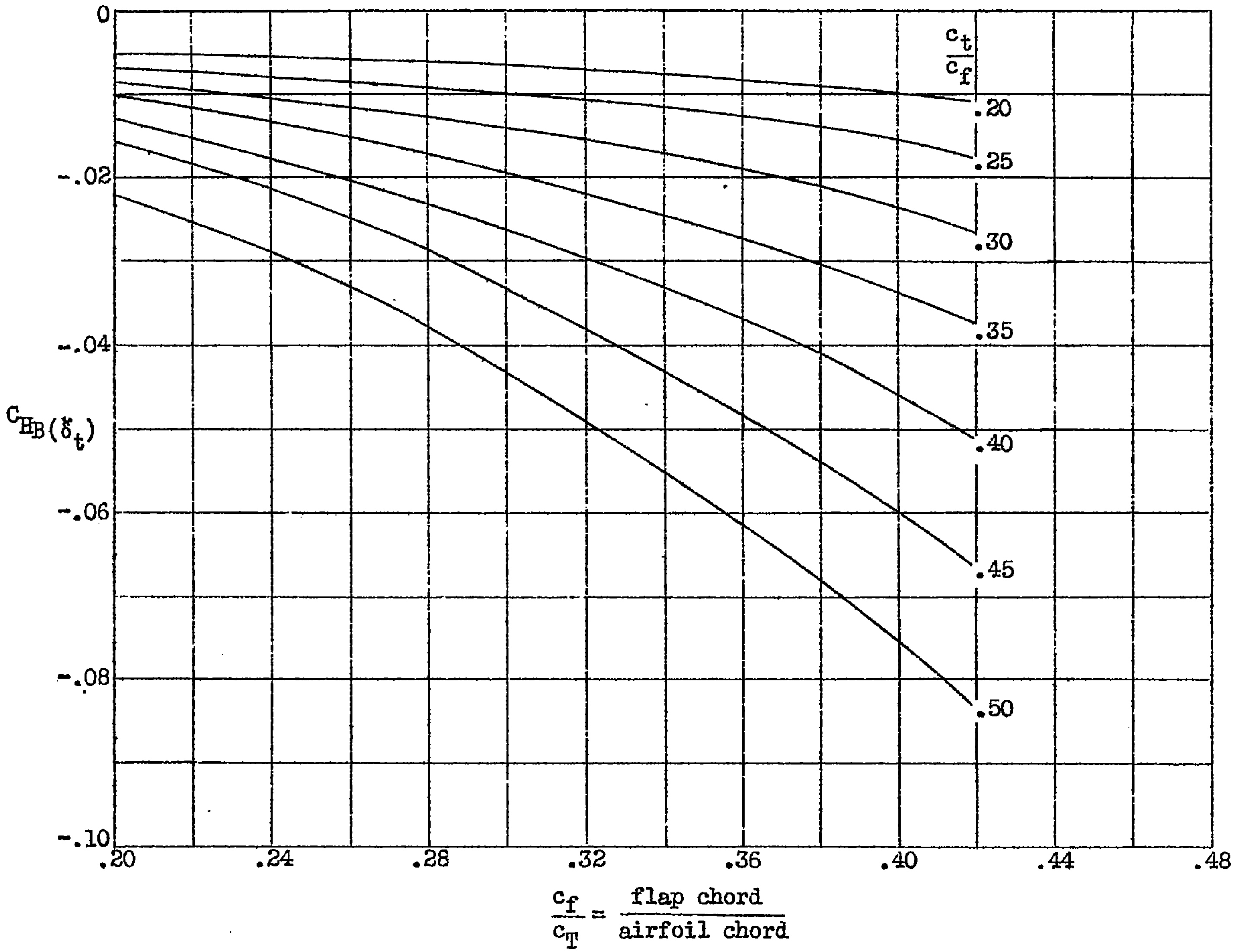

Figure 19.- Basic elevator hinge moment derivative, $C_{H_B(\delta_t)}$ for tab changes (per radian).